

Costas Arrays to Order 1030

IEEE DataPort Database

DOI: <http://dx.doi.org/10.21227/H21P42>

Instructions

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*Instructions for downloading and setting up a searchable database of Costas
arrays to order 1030*

Instructions for the use of the Costas array database analysis and extraction utility

Release 02, January 13, 2018

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1 Files in the Database

1.1 Data Files

This IEEE DataPort database contains Costas arrays for orders from 3 through 1030. For each order, either five (folder\Searches) or nine (folder\Generated) files are included for each order. These files are summarized in Table 1.

Table 1. Files in Database

File name	Purpose	Contents
Sorted_Costas_arrays_N=<order>.txt	Comprehensive list	All CAs
Costas_essense_N=<order>.txt	Shortest list	One Costas array for each set of four or eight
Costas_symmetrical_N=<order>.txt	List of all symmetrical CAs	All symmetrical CAs
Costas_G-Symmetrical_N=<order>.txt	Glide-symmetrical list	Glide-symmetrical CAs
Costas_log_N=<order>.txt	Summary information	Data for extraction utility
Costas_essense_N=<order>_methods.txt	Generator information	Hex 32-bit masks
Costas_G-Symmetrical_N=<order>_methods.txt	Generator information	Hex 32-bit masks
Costas_symmetrical_N=<order>_methods.txt	Generator information	Hex 32-bit masks
Sorted_Costas_arrays_N=<order>_methods.txt	Generator information	Hex 32-bit masks

The generators in reference [2] are used for orders 3 through 1030 to generate the files that are listed in Table 1. All these files must be placed in the folder /Generated for the database utility to work seamlessly for all orders.

For orders that have been comprehensively searched, the files are placed in folder /Searches; included are orders up to 29, which have comprehensive searches in the peer-reviewed literature^{1,2,3,4,5,6}. Since the “*_methods.txt” files are produced by the generator program and have no relevance when the Costas arrays are found by searching, there are no files of the form “*_methods.txt” in the /Searches folder.

Here, we represent Costas arrays of order N as a set of N integers. If these integers are considered as elements of a row vector, each integer is the row index of a 1 in the corresponding column of a matrix, all other elements being zero; the resulting matrix is a Costas array. This representation is not unique; the notation or convention of this sort determines the actual permutation matrix orientation. Different conventions will result in Costas array matrices that differ from other conventions as the transpose or in having the rows or columns reversed, or a combination of these transformations.

Since rotation, transposition, or reversing the order of the rows or columns of a Costas array results in another Costas array, sets of Costas arrays related by these transformations are eight Costas arrays if there is no symmetry about the main diagonal or antidiagonal, or four Costas arrays if such symmetry does exist. We call these *congruence sets* because Costas arrays in a given such set are congruent on transformations of transposition and rotation or reversal of rows or columns.

The number of available Costas arrays as a function of order is shown in Figure 1. curves are shown for all Costas arrays, and for generated-only. The equation fit is

$$\langle \text{Total number of Costas arrays up to a given Order} \rangle \approx 0.154 \cdot (\text{Order})^{2.82}$$



Figure 1. Total Costas Arrays Up To a Given Order

1.2 Database Utility Executable

There is a database utility in the Scripts section of this DataPort entry as the file Read_CA_Database_00.zip. This is a Windows GUI program as a 64-bit binary, accompanied by several DLLs that provide support for Qt5 widgets. If your computer does not have a compatible version of Qt5 installed, place these DLLs in the same folder as the EXE file.

This utility is capable of reading the entire database or any portion of it, as determined by user inputs on a pop-up dialog. It has several modes; the user invokes a particular mode by selection of a menu item or a hotkey. The main application is provided as a scrolling text window.

Modes are provided for simple database extraction according to user-specified criteria (order, symmetry, etc.), database analysis, and Help.

The database utility is, at present, a beta release. The Help utility brings up the current page on this utility on the author's web site; a dedicated Help page for this utility will be used for the first non-beta release, along with speed improvements and evolution of the user interface.

1.3 Other files

The file CA_Database_Information_Files.zip contains hugely helpful files, such as a spreadsheet that provides enumeration data on searched and generated lists including breakdowns by generator. In addition, a small text file produced as an information aid by the generator program provides a summary of the generators. A PDF of the CISS paper that introduced the first version of this database, to order 200⁷, is included.

An auxiliary file, FrHop_LUB_Database.zip provides a list of the LUB (minimum across the set for each order) of differences in integers in the representation of Costas arrays in the database. In the convention given in Section 1 above, if each entry in the Costas array vector represents a time increment in a frequency-hop waveform, and the row index is used for the frequency for that time interval, the difference between successive row indices is proportional to the frequency hop between time intervals. This information may be useful in selection of Costas arrays when PLL-based waveform generators are used.

Additional files that provide different analysis aids for different applications, including mathematical developments, will be added as appropriate.

1.4 Install Only What you Need

The minimum files that you will need to do most extraction and analysis of Costas arrays are these:

- Read_Database_00.zip, 9,047,818 bytes; software utility to read and analyze the database
- CA_Database_Information_Files.zip, 707,300 bytes; enumeration and other information files
- Costas_folder_Searches.zip, 2,417,331 bytes; database for folder /Searches to order 29
- CA_Database_3-100.zip, 8,929,136 bytes; database for folder /Generated to order 100

This is a total of 21.1 MB, easily downloaded in a few seconds. The database extraction and analysis utility will work with any size database automatically. Unpacking database files for orders over about 300 results in unpacked database sizes about three times larger than the ZIP file. Downloading orders beyond 100 and unpacking them into the /Generated folder results in total unpacked database size increase according to about the fourth power of the order, as shown in Figure 2.

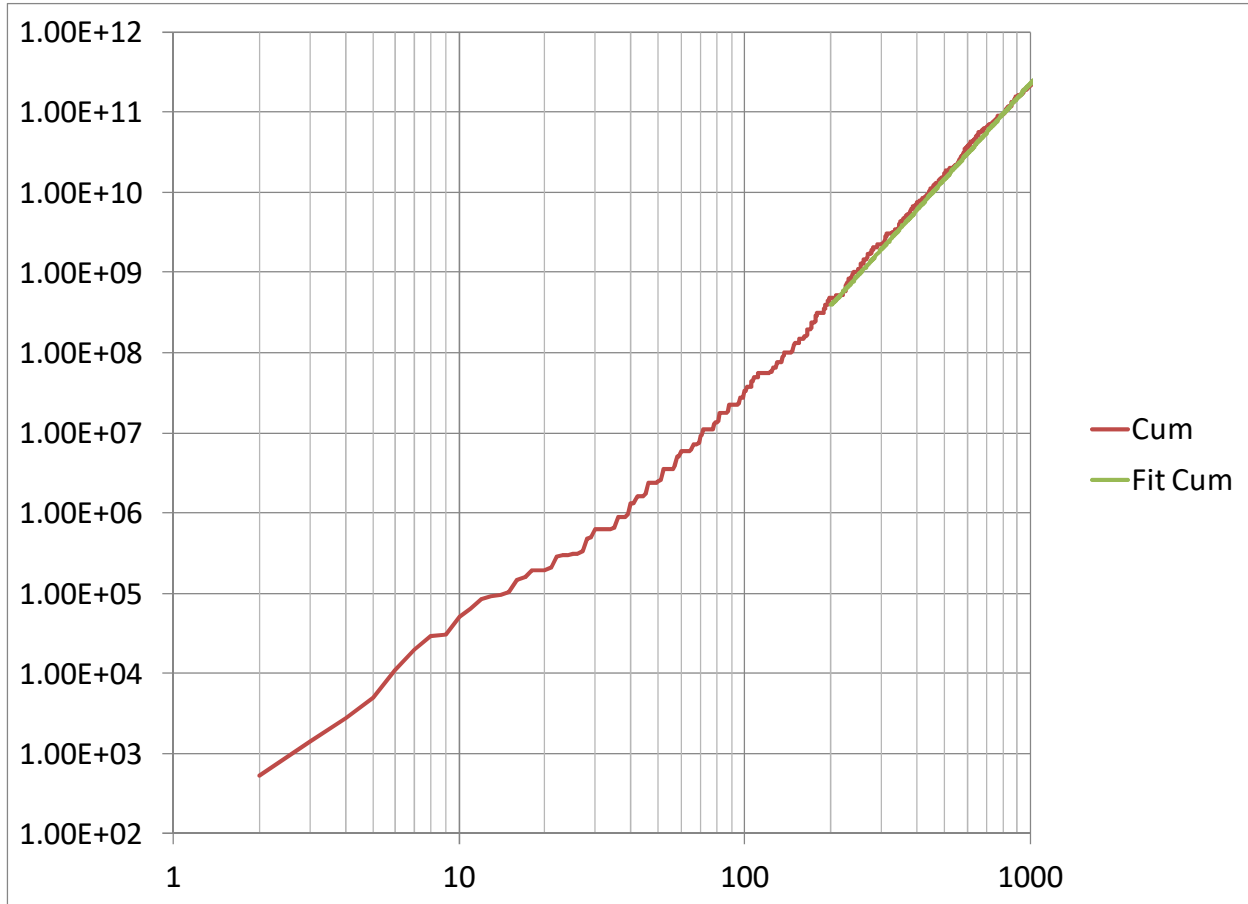


Figure 2. Total Unpacked Size of Database in Bytes versus Maximum Order

The equation for the “Fit Cum” line in Figure 2 that closely models the cumulative database size for orders greater than 200 is

$$\langle \text{Cumulative Size} \rangle \approx 0.28512 \cdot (\text{MaxOrder})^{3.9667}$$

The total file size exceeds 1 GB for orders 255 and over. If your work with Costas arrays requires working directly with orders up to a given order, and the enumeration data in CA_Database_Information_Files.zip is sufficient, then you need only the four files suggested above. Download any more that you need for direct access to Costas arrays of the order you need for additional files for the /Generated folder.

2 Installation of the Database

The Costas array database is contained in the files in Table 2. Extract the files and place in the folders according to Figure 3. When you are done, the result will look like Figure 4. Note that the enumeration spreadsheet file has been placed in the database root folder for convenience. The database extraction and analysis program ignores files in the database root folder.

Table 2. Database Archive Files

Downloaded File	Files	Store in
Costas_folder_Searches.zip	First 5 rows of Table 1	CA_Database/Searches
CA_Database_3-100.zip	All rows of Table 1	CA_Database/Generated
CA_Database_101-200.zip (optional)	All rows of Table 1	CA_Database/Generated
CA_Database_201-300.zip (optional)	All rows of Table 1	CA_Database/Generated
...	All rows of Table 1	CA_Database/Generated

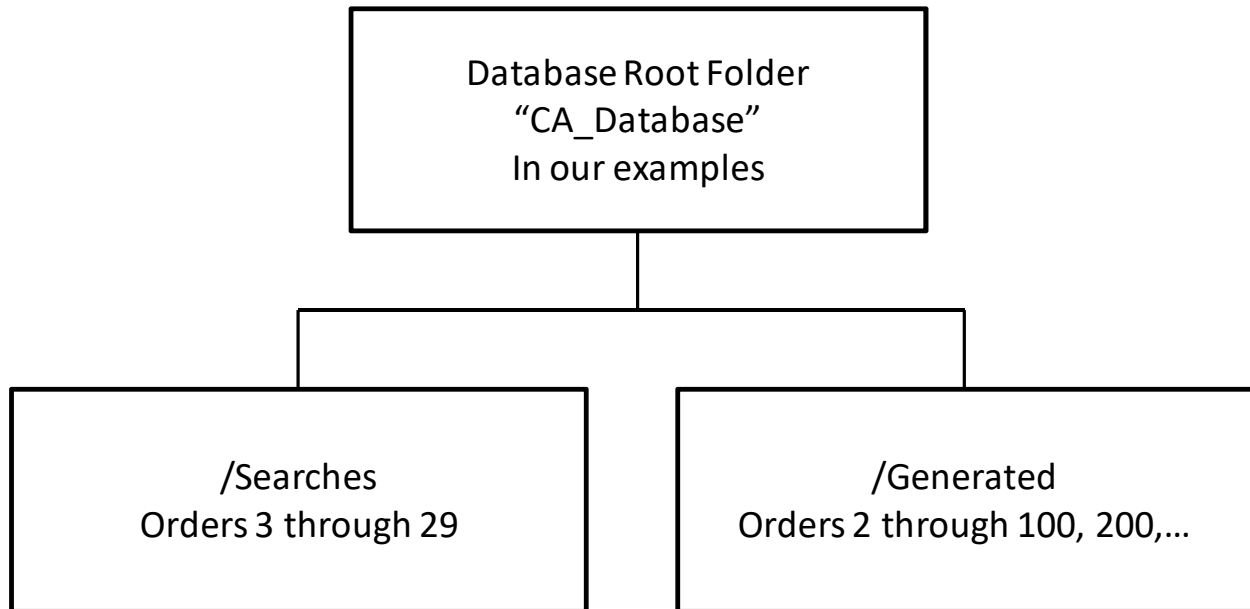


Figure 3. Installed Database Directory Structure

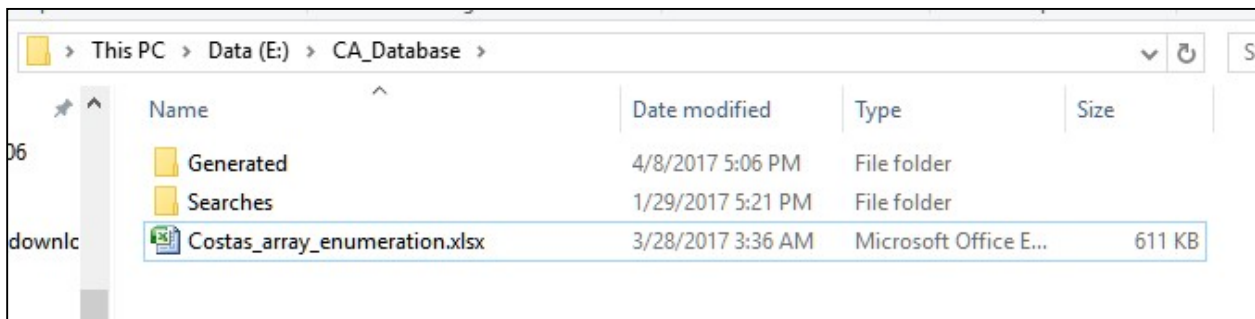


Figure 4. File Manager View of Database Root Folder

The database consists of only the data files shown here. The Windows registry or other system capabilities are not involved beyond defaults, such as association of files with names ending in .txt with applications that read or write text files.

3 Installation and Use of the Costas Array Extraction and Analysis Utility

3.1 Version

The version of the utility documented here is Beta 0.9. The version is part of the initial scrolling text in the base application window and is presented other places as the program executes. You can check for a later version with a hotkey as documented below. This documentation will be updated as new versions are uploaded.

3.2 Downloading and Installation

The file is CA_Database_00.zip, in the Scripts area of the IEEE DataPort page on this database. Note that another copy of CA_Database_00.zip is in the Files area as a work-around for an error in downloading from the Files area.

There are a number of ZIP archive utilities. The screen of Bit Zipper when CA_Database_00.zip is loaded, is shown in Figure 5 below.

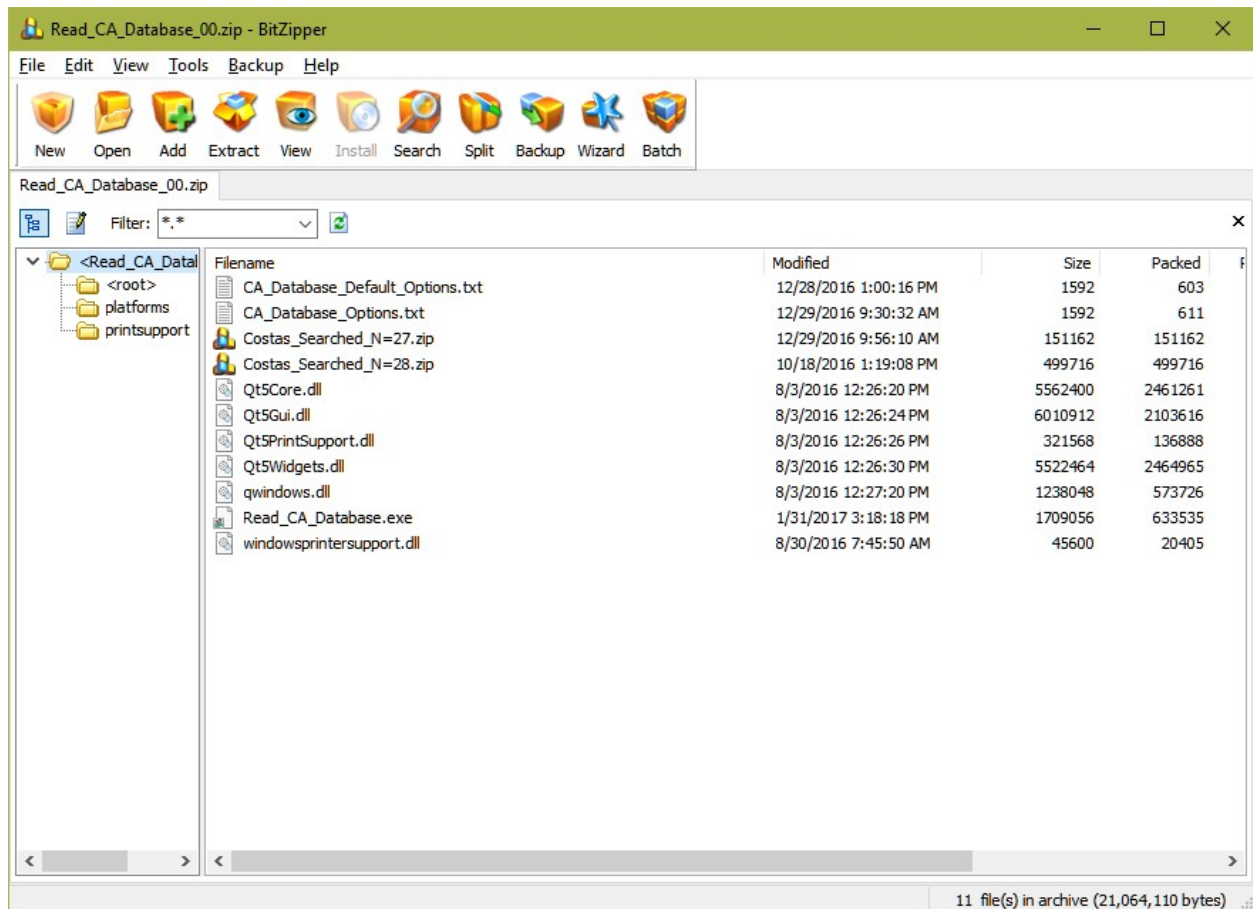


Figure 5. Bit Zipper with CA_Database_00.zip Loaded

Extract the files to any folder, preferably one that is empty (except possibly for CA_Database_00.zip itself). The folder should be as shown in Figure 6 below. The folder /platforms contains qwindows.dll and the folder /printsupport contains windowsprintersupport.dll. These and the DLLs of the form

Qt*.dll are Qt DLLs that support Qt widgets, which are used extensively by the application, Read_CA_Database.exe. The two text files support sticky options (see below) and may be viewed but are maintained by the application and should not be edited. If the program gets an error in reading either of these files, it will replace them with a default and cause inconvenience by forcing the user to restore all the sticky options that the program would ordinarily maintain as the defaults for some options whenever it executes.

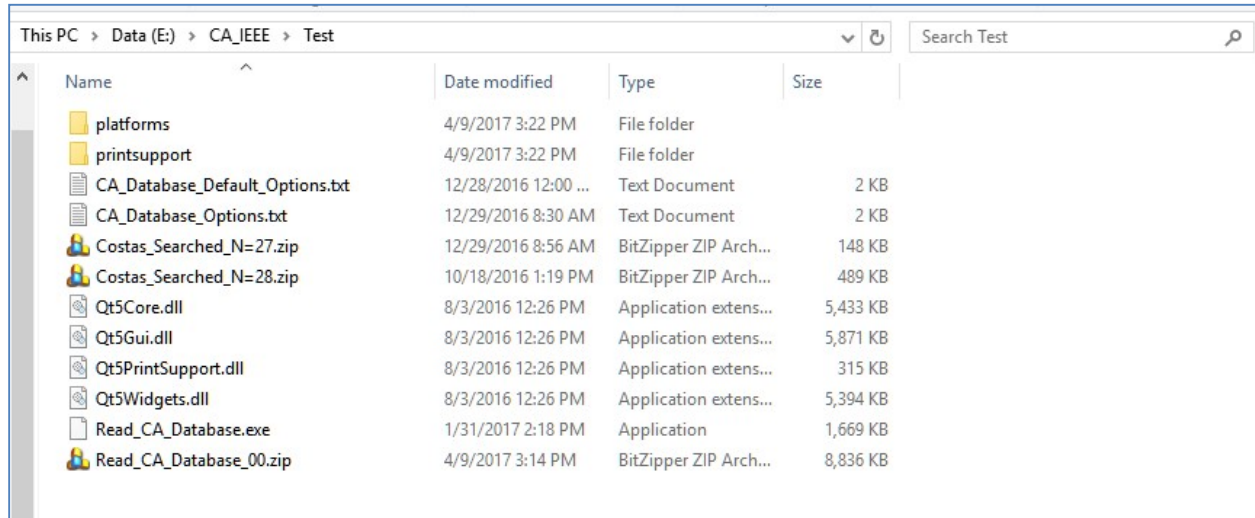


Figure 6. Folder with contents of CA_Database_00.zip

The two ZIP archives Costas_Searched_N=27 and Costas_Searched_N=28 are for the convenience of users with old downloads and should be ignored, and can safely be deleted. They may safely be deleted and will not be included in future releases.

The original download, Read_CA_Database_00.zip, can be deleted once proper execution of Read_CA_Database.exe is verified.

The registry is not used in installation or execution of this program. You can completely delete this application by deleting the folder with the EXE file and the Qt DLLs.

You may now place shortcuts to Read_CA_Database.exe in your user interface. The starting folder should be left blank so that the program starts in the folder with the EXE file; this is because the application-maintained configuration file must be in the folder in which the application is running.

3.3 The Application-Maintained Configuration File

A number of options, once set, are usually not changed again, or at least not changed often, for a particular user. Examples are location of the database files, path for the output files, whether the utility is being used in a Windows or Linux environment (currently Linux and Mac versions are not provided), etc. These options are stored as a text file, CA_Database_Options.txt, after each database mode is executed, and this file is loaded by each database mode as the defaults for some options.

If CA_Database_Options.txt is not present, or if the program encounters a read error in loading this file, the file CA_Database_Default_Options.txt is copied to CA_Database_Options.txt. If there is a read error for CA_Database_Default_Options.txt, it is replaced by the program, but the user may edit this program to change the default options.

4 Using the Utility

4.1 Starting the Utility

When first executing Read_CA_Database.exe, the license file dialogue pops up. This window is shown as Figure 7 below.

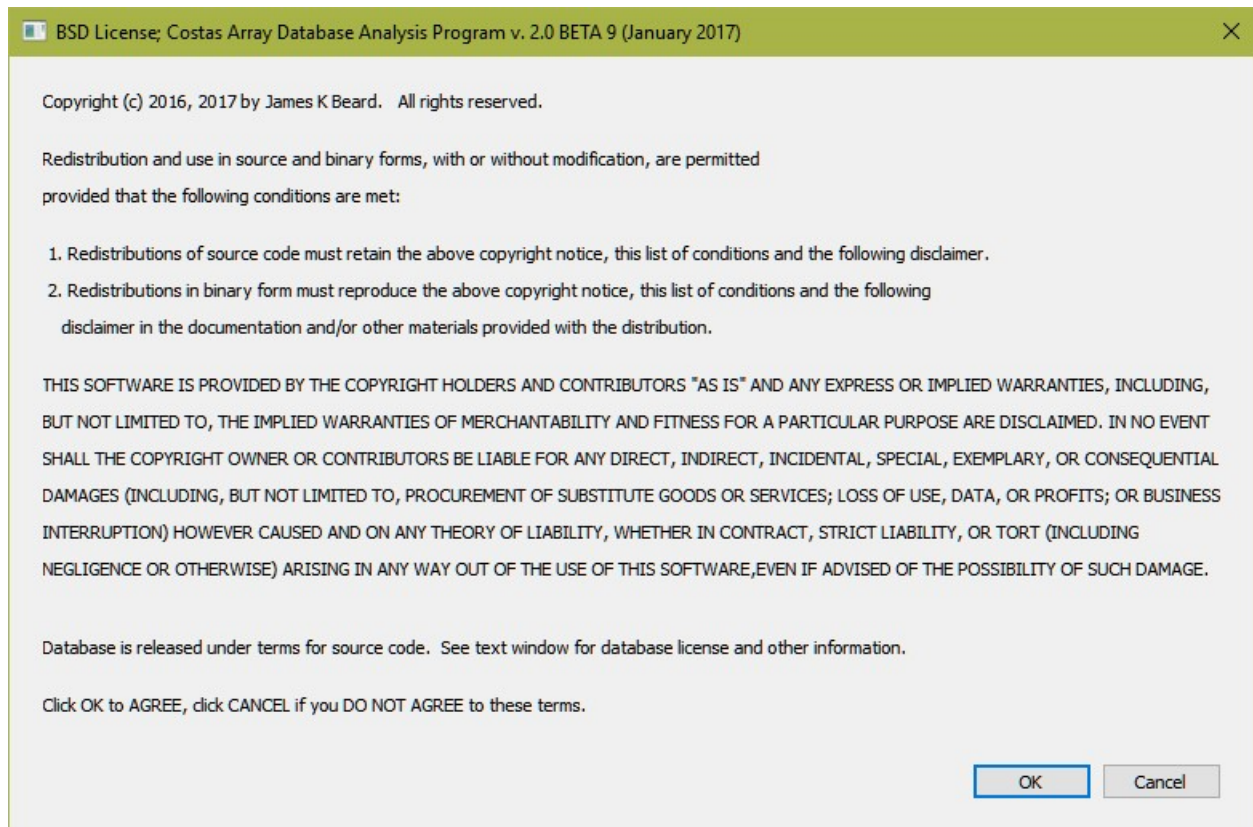


Figure 7. License Dialogue

Note that this is the Simplified BSD license, available from the Open Source Initiative at <https://opensource.org/licenses/bsd-license.php>. But, note that **if you downloaded this file from the IEEE DataPort**, its use is governed instead by the Creative Commons Attribution (BY) license. The Creative Commons Attribution license is available at <https://creativecommons.org/licenses/by/4.0/>. A complete move to the Creative Commons Attribution license will be implemented in future releases of the files. Both licenses allow use and extension of the work, including incorporation of the work into other work, including free and commercial software, with the only requirement being attribution, and you may not claim ownership to the database or restrict access or use of the database by others. Examples of attribution:

[1] James K Beard, "Costas arrays and enumeration to order 1030", IEEE Dataport, 2017. [Online]. Available: <http://dx.doi.org/10.21227/H21P42>. Accessed: <date you downloaded>.

BiBTeX:

```
@misc{h21p42-17,  
doi = {10.21227/H21P42},  
url = {http://dx.doi.org/10.21227/H21P42},  
author = {Citation Author(s): James K Beard },  
publisher = {IEEE Dataport},  
title = {Costas arrays and enumeration to order 1030},  
year = {2017} }
```

Note that the DOI link will take you to the database pages in IEEE DataPort, which may be accessed by anyone with an IEEE logon. On that page, there is a box below the AWS S3/Download/Scripts box as shown as Figure 9. Pressing the Cite button brings up the citation formatting window shown here as Figure 8. Clicking the tabs brings up clipboard-copyable examples that reference this database according to the format. The two examples given above are the IEEE format and the BibTeX format; note that the [bare_jrnl.tex](#) template by Michael Shell accepts the label @misc{<label>,... rather than @data{<label>,... as given in the Cite dialogue. This template loads [IEEEtran.cls](#).

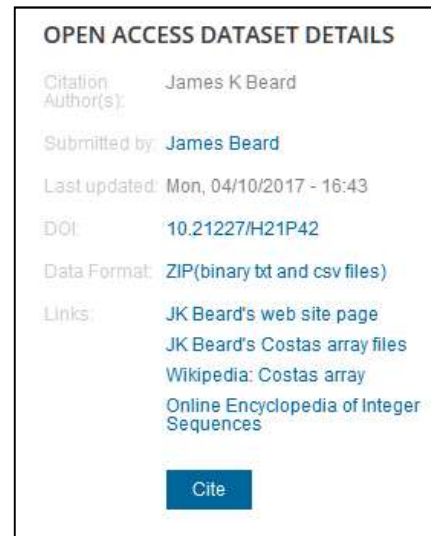


Figure 9. Box Including "Cite" Button

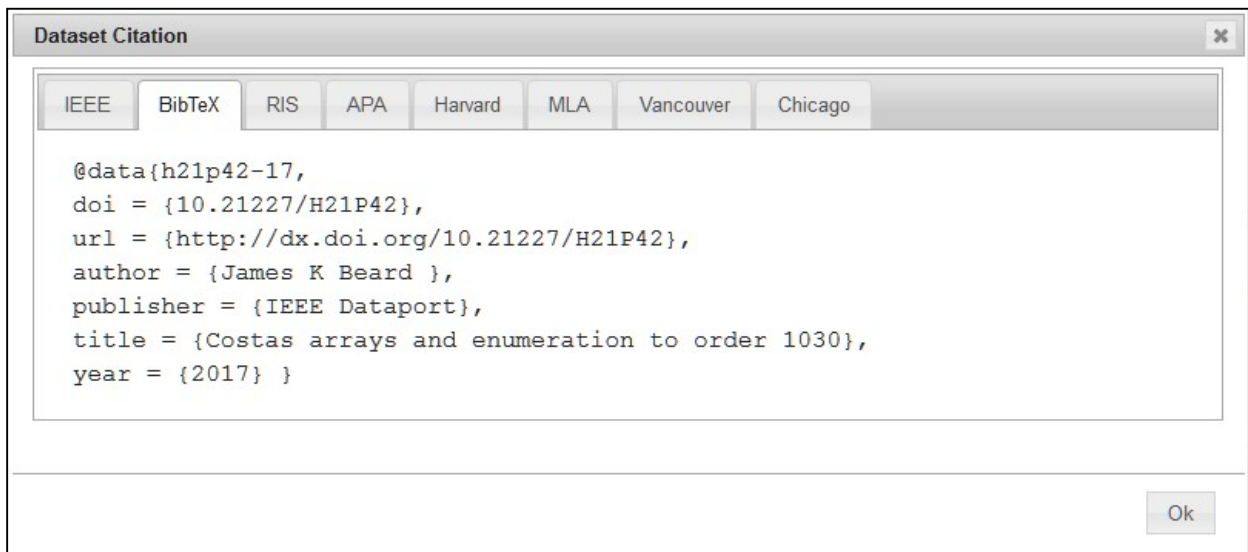


Figure 8. Citation Window Showing BibTeX Format

4.2 The Opening Screen

Clicking OK on the license dialogue opens the screen shown in Figure 10. If Cancel is clicked, only the first line with the version number is printed, the Program menu is not present, and the hotkeys are not enabled, and no action is possible other than to close the window and try again.

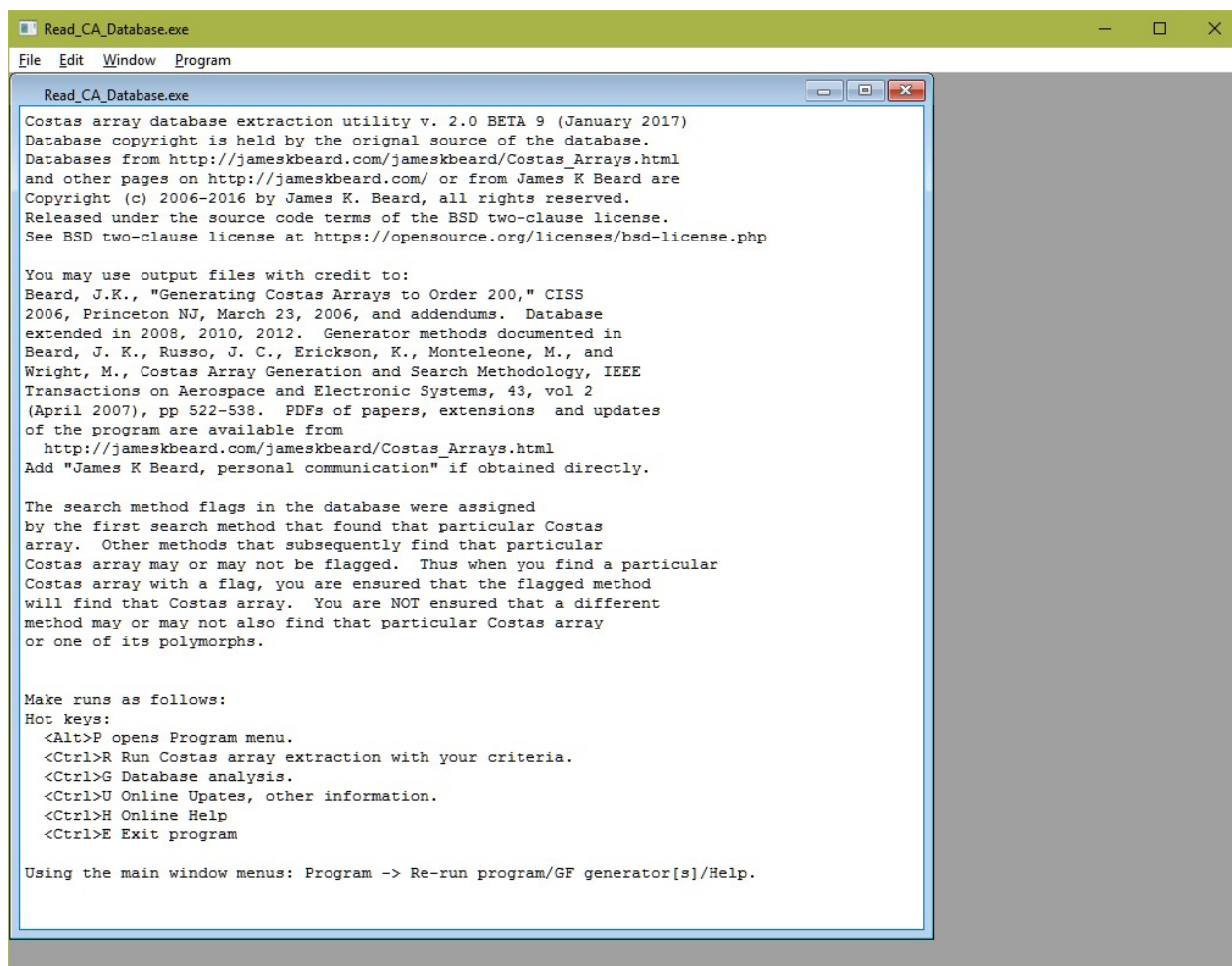


Figure 10. Opening Screen

The text does not reflect the existence of the IEEE DataPort hosting or the Creative Commons Attribution license; this will be corrected in the next release.

4.3 Mode Selection

From the last few lines of text in Figure 10 show hotkeys that may be used to open the program menu, which brings up the pop-up shown in Figure 11, or the hotkeys that may be used instead. The Online modes open web pages and require Internet access to function properly. The extraction mode performs selection on a user-stipulated order of Costas array,

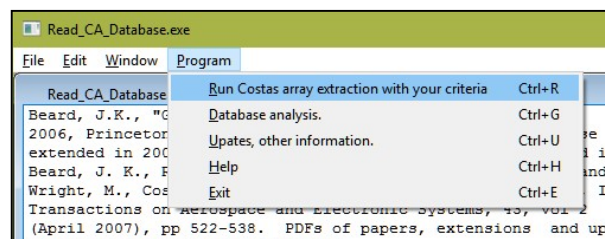


Figure 11. Program Menu

while the analysis mode operates over a range of orders, not just one order.

4.3.1 The Background Scrolling Text Window

The application itself runs in the main application window with the application title, and a scrolling text window runs in this window, as shown in Figure 10. Closing the application window closes the application. Closing the text window is possible but we advise against that because status, information, and heartbeat information are posted to the scrolling text window. Although the Costas arrays that are selected by use of the application are written to a CSV file as output, the scrolling text window allows preview of the first 20 terms in any Costas array selected for output.

4.3.2 Selecting and Invoking a Mode

Invoking a mode or exiting the program may be done through the Program menu, or by using <Alt>P to open the Program menu and pressing the underlined letter. The application may be terminated by closing the application menu, pressing <Ctrl>E, or through the Program menu by Program->Exit.

4.4 The Selection and Analysis Modes

Invoking the Selection mode by <Ctrl> R or the Program menu brings up the dialogue shown in Figure 13. Invoking the Analysis mode by <Ctrl> G or the Program menu brings up the dialogue shown in Figure 14. The dialogues are similar, each having three sections, Program Configuration, Extraction, and Computation. Configuration options determine how the program operates; these options are not changed often, and are “sticky” options that default to the values used in the most recent run. Extraction options provide several ways that you can filter the Costas arrays to be extracted or used in computations. Computation options are operations that are to be performed on Costas arrays that meet the filtering criteria in the Extraction section. The overall dialogues seem similar, with the differences being

- The Selection mode accepts only one Costas array order, whereas the Analysis mode accepts a starting and ending order, and repeats its operations over a range of orders.
- There are a lot more options available in the Computation section of the Analysis mode.

In the following sections, we will walk through the options in each section of these dialogues.

4.4.1 Program Configuration

4.4.1.1 Location of Database

The database itself may be on a folder on your system drive, a folder on another hard disk or USB hard disk, on a USB thumb drive, a CD-ROM or data DVD, or a network drive that is attached and available to applications as a drive letter. The database root folder must be known to the application, and is the first option in each mode that accesses the database. The database root folder must contain the subfolders /Searches and /Generated; see Figure 3 for a file manager view of the database root folder.

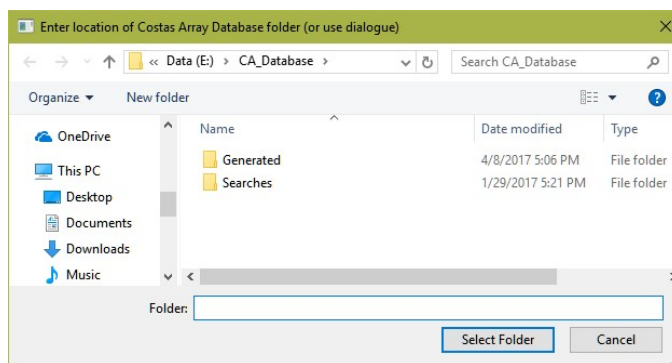


Figure 12. The Database Folder Selection Dialogue

Entering the location of the database root folder is done through a file manager interface as shown in Figure 12.

4.4.1.2 Generated Database Only Checkbox

The database is in two folders, /Generated and /Searches. The /Searches database contains databases on orders 2 through 29. These orders have been subjected to exhaustive searches in the peer-reviewed literature^{1,2,3,4,5,6,7}. For orders 8 through 27, the exhaustive searches find Costas arrays that are not produced by the generators⁶. For this reason, to make all known Costas arrays available to the program, the recommended state is to use the /Searches database for orders 27 and below. Although orders 28 and 29 have been exhaustively searched, there are none that are not produced by the generators^{4,5}. The /Generated database includes information about which generator produces each Costas array, so that database is preferred, and is always used for orders 28 and higher.

When only the Costas arrays that are produced by the generators are desired, or when generator information is essential, the user may check this box and the data in /Generated which includes the information on which generator produces each particular Costas array.

4.4.1.3 Index from 0 to N-1 Checkbox

The program produces comma-separated values (CSV) files for use as input to other programs or loading by spreadsheet programs, and for output to the scrolling text window. The user or software that uses these files may prefer that the Costas array row vector be indexed from 0 to N-1, and this is done when this box is checked. Default when this box is not checked is to index from 1 to N.

v. 2.0 BETA 9 (January 2017) Parameters for This Run

PROGRAM CONFIGURATION

Database folder must contain the folders "Generated" and "Searches"

Enter location of Costas Array Database folder (or use dialogue) ...

☐ Check to use generated database only, Uncheck to use searched database to order 27 then generated to order 1030

☒ Check to overwrite existing output files, uncheck to append to them.

☐ Check to index from 0 to N-1; uncheck to index from 1 to N

PARAMETERS FOR EXTRACTION OF COSTAS ARRAYS

Enter order of Costas arrays:

Select Generator Method, if any (ignored for Searches database)

Select Costas array type

Select maximum number of ones in central 3X3 (4 to not filter):

Select maximum number of ones in central 5X5 (16 to not filter):

Select option on filtering by maximum frequency hop

☐ No filtering by maximum frequency hop.

☐ Select only Costas arrays that have the minnum peak frequency hop, within the Costas array.

☒ Select only Costas arrays that have the minnum peak frequency hop, end-around.

COMPUTATION OPTIONS

☒ Check to provide a DAF for each Costas array, output to individual CSV files.

☐ Check to find a polynomial fit in GF(q). If checked, please provide estimate of q below.

Enter q for Galois field; search for viable value begins with your entry here:

Figure 13. The Selection Mode Dialogue

v. 2.0 BETA 9 (January 2017) Parameters for This Run

PROGRAM CONFIGURATION

Database folder must contain the folders "Generated" and "Searches"

Enter location of Costas Array Database folder (or use dialogue)

☐ Check to use generated database for all orders, Uncheck (RECOMMENDED) to use searched database to order 27 then generated to ord

☒ Check to overwrite existing output files, uncheck to append to them.

☐ Check to index from 0 to N-1; uncheck to index from 1 to N

SELECTION AND FILTERING PARAMETERS FOR EXTRACTION OF COSTAS ARRAYS

Enter starting order of Costas arrays:

Enter ending order of Costas arrays:

Select Generator Method, if any (ignored for Searches database)

Select Costas array type

Select maximum number of ones in central 3X3 (4 to not filter):

Select maximum number of ones in central 5X5 (16 to not filter):

Select option on filtering by maximum frequency hop

☐ No filtering by maximum frequency hop.

☐ Select only Costas arrays that have the minnum peak frequency hop, within the Costas array.

☒ Select only Costas arrays that have the minnum peak frequency hop, end-around.

COMPUTATION OPTIONS

☒ Check to provide a DAF for each Costas array, output individual CSV files.

☐ Check to find a polynomial fit in GF(q). If checked, please provide estimate of q below.

If previous box checked, enter q for Galois field; search for viable value begins with your entry here:

☐ Check to write CSV file of counts of different types and database file sizes.

☐ Check to output CSV of counts of ones in central 3X3 square (SLOW for lots of large orders)

☐ Check to output CSV of counts of ones in central 5X5 square (SLOW for lots of large orders)

☐ Check to re-compute minimum peak frequency hop database . Orders taken from database; SLOW for lots of large orders.

☐ Check to output DAF max and min CSV (SLOW for lots of large orders).

OK
Cancel

Figure 14. The Analysis Mode Dialogue

4.4.2 Filtering Parameters

4.4.2.1 Order of Costas Arrays (Selection Mode only)

In the Selection mode, you enter the order of Costas arrays that you are investigating here.

4.4.2.2 Starting and Ending Orders of Costas Arrays (Analysis Mode only)

In the Analysis mode, you enter the starting and ending orders of Costas arrays that you wish to investigate here. They may be the same order if you are only looking at one order, but you want the analysis options available in this mode. If the ending order is lower than the starting order, no action is performed.

4.4.2.3 Generator Method Combo Box

The Generator Method combo box allows the user to select only one generator method. About half of the list is shown in the screen shot of Figure 15. The full list is shown in Table 3 from [2]. No filtering by method is done when the default, All methods, is selected. This option is grayed out when data is taken from a searched database and method information is not available.

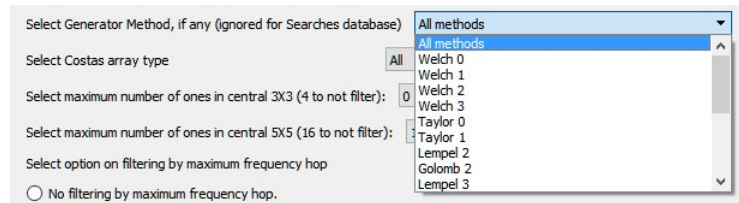


Figure 15. Generator Method Combo Box

Table 3. Generator Methods

Method	Notes
Welch 0	Welch 1, add dots at one of (1, 1), (1, p), (p, 1), or (p, p)
Welch 1	One of two base methods, $i = \alpha^{j-1} \in \text{GF}(p)$, p prime
Welch 2	Welch 1, with dot at (1, 1) removed, when it is present
Welch 3	Welch 1 with dots at (1,1),(2,2) removed; these dots are present when $\alpha = 2$
Taylor 0	Lempel-Golomb 2, when $q \equiv -1 \pmod{6}$ add dot pair at (1, 1), (q, q) or (1, q), (q, 1)
Taylor 1	Lempel-Golomb 2, add a dot at (1, 1), (1, q), (q, 1), or (q, q)
Lempel-Golomb 2	One of two base methods, $\alpha^i + \beta^j \equiv 1 \in \text{GF}(q)$
Lempel-Golomb 3	Lempel-Golomb 2, dot at (1,1) removed, works when $\alpha = 2$
Lempel-Golomb 4	Lempel-Golomb 2 when $q = 2^k$, dots at (1, 1) and (2, 2) removed
Taylor 4	Lempel-Golomb 2, $\beta = \alpha$ (Lempel), dots at (1, 2) and (2, 1) removed
Golomb* 4	Lempel-Golomb 3, $\alpha + \beta = 1$, delete dots at (1, 1), (2, q - 2)
Golomb* 5	Golomb* 4 with additional dot at (q - 2, 2) also removed
Inhom. Add 1	Lempel-Golomb 2 with γ on right-hand side; singularity
Inhom. Sub 1	Inhomogeneous Additive 1, but with subtraction on left-hand side; singularity
Rickard W 0	Welch 1, j incremented mod(p), with added dot
Rickard L-G 1	Lempel-Golomb 2, j incremented mod(q - 1), added dot

The generator program sets a bit in a bitmask at a position that denotes one of the generators in Table 3 when a Costas array is generated and is found not to be a duplicate of a Costas array already in the

database. Then, the Costas array is transposed and rotated to find all four or eight Costas arrays in its congruent set and all four are added to the database with the same method bitmap. Careful examination of this scheme reveals that it is possible that Costas arrays are in the database that are generated by methods that are not flagged in the associated methods bitmask.

The generator routine, not released to anyone at present, is capable of performing searches using any combination of methods, and one can be assured that all the Costas arrays found by a particular method may be found in a run focused on that determination by enabling only that method.

The methods in Table 3 all begin with a Welch 1 generator, which generates Costas arrays of order one less than every prime, and the Lempel-Golomb 2 generator, which generates Costas arrays of order two less than any power of a prime $N + 2 = q = p^k$. Note that each uses a Galois field of order q , denoted as $GF(q)$. Positive integer arithmetic modulo a prime p is the simplest example of arithmetic in a Galois field. Several of the generator methods apply only for such fields, i.e. $q = p^1 = p$. In particular, the Welch generator and any of the methods based on it apply only with $k = 1$. The Lempel-Golomb method applies for $k > 1$ but the Taylor 0, Lempel-Golomb 3, Taylor 4, Golomb*4 and Golomb*5 all begin with $GF(p)$ or $k = 1$.

4.4.2.4 Costas Array Type Combo Box

The Costas array type combo box allows filtering for symmetrical Costas arrays, or only searching a minimal database that contains just one of the four or eight members of any given congruent set. This combo box is shown in Figure 16.

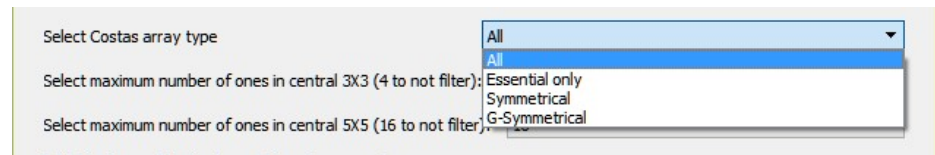


Figure 16. Costas Array Type Combo Box

The list of Essential Costas arrays is found by selection of the one of the four or eight Costas array vectors in each congruent set that begins with the smallest column indices.

The list of Symmetrical Costas arrays includes only those Costas arrays of a given order that are symmetrical about the main diagonal. Note that all Costas arrays generated by the Lempel 2 method, e.g. Lempel-Golomb with $\alpha = \beta$, are symmetrical, as are generalizations that begin with such arrays when adding or deleting dots preserves the symmetry property.

1									
	1								
			1						
							1		
				1					
									1
								1	
						1			
		1							
					1				

Figure 17. G-Symmetric Costas Array of Order 10

The list of G-symmetrical Costas arrays is a list of glide-symmetric Costas arrays. These are enumerated and given their own section in each database because Herbert Taylor⁸ noticed that all Costas arrays generated by the Welch 1 method had glide-symmetry, which he defined for Costas arrays as the bottom half of the matrix being congruent with the top half, but with the order of the rows reversed. Figure 17 shows order 10 Costas array (1,2,9,3,5,10,8,4,7,6). Note that the top and bottom five rows are congruent on reversal of rows. Entries in the G-Symmetric database are determined by checking for this property, not by selecting Costas arrays generated by the Welch 1 method.

				1								
									1			
1												
											1	
					1							
		1										
						1						
								1				
			1									
											1	
	1											
							1					
									1			

Figure 18. Order 13 G-Symmetric Costas Array

An interesting logical point is raised for odd order G-symmetric Costas arrays: There will be a middle row, and for G-symmetry to apply the middle row must have a 1 in the center element. But since the Welch 1 generator generates Costas arrays of order one less than a prime, then there will be none generated that are of odd order. But with the database extraction utility we find that there are 16 G-symmetric Costas arrays of order 13; the example (3,11,6,9,1,5,7,12,8,2,13,4,10) is shown as Figure 18, and, yes, the center element is a one. Our enumeration spreadsheet in CA_Database_Information_Files.zip tells us that these are Lempel-Golomb 3 Costas arrays, and, further, that four G-symmetrical Costas arrays of order 5 exist. One of these is shown in Figure 19; our enumeration spreadsheet tells us that this Costas array may be generated by Lempel-Golomb 3 or other methods.

1				
			1	
		1		
				1
	1			

Figure 19. Order 5 G-Symmetric Costas Array

Taylor and others have performed comprehensive searches over orders higher than 29 with the constraint of symmetry and G-symmetry; he and others have reported⁹ that there are no symmetrical or G-symmetrical Costas arrays of order 32 or 33.

4.4.2.5 Maximum Number of Ones in Central 3X3 (4 to not filter)

In formulating a waveform, selection may be done in part to achieve better performance near the central peak of the ambiguity function¹⁰.

This option and the next offer an opportunity to select Costas arrays with a discrete ambiguity function (DAF) that has not only a specified number of ones near the central peak, which is always equal to N. The DAF is defined from a stationary Costas array matrix as shown in Figure 17, Figure 18 or Figure 19, and an identical Costas array that is moved to the right/left or up/down by a discrete number of rows and

columns. By the Costas condition, the count is always zero or one except with zero shifts, when the count is N .

The properties of a convolution matrix make it impossible for any ones to coincide when the number of rows shifted is zero except when the number of columns shifted is also zero, and vice versa. Therefore the maximum number of ones possible in the 3X3 central square of the DAF is $(3-1)^2 = 4$, not $3^2 = 9$, so setting this threshold at 4 disables filtering. The DAF is glide-symmetrical so this option is a combo box with allowable values of 0, 2, and 4.

4.4.2.6 Maximum Number of Ones in Central 5X5 (16 to not filter)

This combo box allows the user to filter on the number of ones allowed in the central 3X3 cells of the DAF. The maximum number of ones possible in the 5X5 central square of the DAF is $(5-1)^2 = 16$, not $5^2 = 25$ and again the number of once must be an even number, so we have a combo box with allowable values of even numbers between zero and 16.

4.4.2.7 Filtering by Maximum Frequency Hop Radio Buttons

Costas arrays are used in frequency-hop schemes where the row index represents a frequency, and, usually, these frequency bins are adjacent and equally spaced. When the user is selecting Costas arrays according to maximum difference between adjacent row indices, the selection minimizes the magnitude of the frequency change required to implement the waveform¹⁰. Although direct digital synthesis effectively makes this frequency change time zero, phase-locked loop (PLL) implementations are often used in small, low-power, or low cost systems such as cell phones, WiFi modems, etc. This radio button allows selection of the Costas arrays of a given order that are the minimum for that order. A second radio button extends the filtering to end-around differences in row index.

4.4.3 Computation Options for All Database Access Modes

4.4.3.1 Provide DAF Checkbox

Checking this box provides the DAF an additional CSV file.

4.4.3.2 Provide Generator Polynomial in $GF(q)$ Checkbox

Checking this box provides a polynomial in $GF(q)$ that generates the Costas array row indices. The generator is of the form

$$x^{r(i)} = \sum_{s=0}^{N-1} c_s \cdot x^{i \cdot s}$$

where x is any generating element of $GF(q)$ and $r(i)$ is the i^{th} row index of the Costas array vector.

The coefficients are stored as integers $\text{index}(c_s)$ so that the coefficients c_s are found by $c_s = x^{\text{index}(c_s)}$ and the index of the value of the polynomial is the row index of a one for column i . This allows the use of integers between zero and $q-2$ to represent all the nonzero elements of $GF(q)$ in a CSV file. A negative integer means that the coefficient is zero.

If this box is checked, you must enter a value for q in the text box immediately below the check box. The value of q must be $N + 2$ or larger. The program checks to see if the entered value of q is a valid value of the form p^k where p is a prime and k is a positive integer; if not, the value of q is incremented by one and the test repeated; this continues until a valid value of q is found for which $GF(q)$ exists. This ensures that a polynomial in elements of $GF(q)$ that generates the Costas array exists.

This capability was used in some work reported in CISS¹¹. Generalizations of the Lempel-Golomb generator were considered, and this utility does reproduce the generator when it is given a Lempel-Golomb 2 Costas array. Modified Lempel-Golomb inhomogeneous and Rickard Lempel-Golomb may also produce generator polynomials with only a few nonzero coefficients. This capability is available for research.

4.4.4 Computation Options Available in Analysis Mode Only

4.4.4.1 Write CSV File of Database File Sizes Checkbox

Checking this box enables an analysis of the database file sizes. It is very slow for large databases and of limited value, because its principal information is given in Figure 2, but the capability is available to the user. A CSV file is written with counts of different types of Costas arrays and of the file sizes in the database.

4.4.4.2 Output CSV of Counts of Ones in Central 3X3 Square Checkbox

Check this box to provide counts of ones in the central 3X3 square of the DAF.

4.4.4.3 Re-Compute Minimum Peak Frequency Hop Database Checkbox

The minimum frequency hop for each order is hard-wired in the database extraction and analysis utility. Check this box to recompute it for your database as stored on your computer.

4.4.4.4 Output DAF Max and Min CSV Checkbox

Check this box to output CSV files that represent the maximum and minimum values in each DAF cell. The DAF is a square matrix $(2 \cdot N - 1)$ cells on a side. Orders with large numbers of Costas arrays will produce a minimum DAF matrix that is mostly zeros except for the center cell (which is N) and a maximum DAF matrix that is mostly ones except for the center cell.

4.5 The Help Screen Mode

This option brings up the web page <http://jameskbeard.com/jameskbeard/Files.html#CostasArrays> in your default web browser. Future versions will look in the database root folder for the current version of this file and display it in the PDF viewer of your web browser. This provides a Help capability for offline computers. If it is not found there, this mode will display this file from the IEEE DataPort web site or my web site.

4.6 Checking for Newer Version

This option looks at <http://jameskbeard.com/jameskbeard/Files.html#GUIDATABASE>, the online posting of the database extraction utility on my web site. Future versions will look for it on the IEEE DataPort web site.

4.7 Future Modes

The modes that exist in the database utility were developed and installed to meet the needs of users as these needs arose. If you have a suggestion for a mode that is of interest to users of Costas arrays, please post your suggestion to the IEEE DataPort page for this database.

5 Other files

5.1 Minimum Frequency Hop Analysis File

The minimum frequency hop, both the simple and the end-around computation, are available as a spreadsheet file.

5.2 Future Analysis Files

More analysis files will be uploaded as available. You may submit files or ideas for new analysis files that you believe are of interest to Costas array users.

¹ J. K. Beard and K. Erickson and M. Monteleone and M. Wright and J. C. Russo, "Combinatoric collaboration on Costas arrays and radar applications", Proceedings of IEEE Radar Conference pp 260-265 (2004).

² J. K. Beard and J. C. Russo and K. G. Erickson and M. C. Monteleone and M. T. Wright, *Costas array generation and search methodology*, IEEE Transactions on Aerospace and Electronic Systems, vol. 43 no. 2, April 2007, pp 522-538 (2007).

³ K. Drakakis and S. Rickard and J. K. Beard and R. Caballero and F. Iorio and G. O'Brien and J. Walsh, *Results of the enumeration of Costas arrays of order 27*, IEEE Trans. Inf. Theory, vol.54 pp 4684-4687 (2008).

⁴ K. Drakakis and F. Iorio and S. Rickard, *The enumeration of Costas arrays of order 28*, Proceedings of Information Theory Workshop, pp 1-5, August/September 2010.

⁵ K. Drakakis and F. Iorio and S. Rickard and J. Walsh, *Results of the enumeration of Costas arrays of order 29*, Advances in Mathematics Communications, vol. 5, pp 547-553 (2011).

⁶ Jon C. Russo, Keith G. Erickson, and James K. Beard, *Costas Array Search Technique that Maximizes Backtrack and Symmetry Exploitation*, 44th Annual Conference on Information Sciences and Systems (CISS) 2010, May 13, 2010, pp 1-8, DOI: 10.1109/CISS.2010.5464772

⁷ James K Beard, *Generating Costas Arrays to Order 200*, 2006 40th Annual Conference on Information Sciences and Systems (CISS), March 23, 2006, DOI: 10.1109/CISS.2006.286635

⁸ Herbert Taylor, *Costas Arrays*, Chapter IV.7 in *The CRC Handbook of Combinatorial Designs*, Charles J. Colbourn and Jeffrey H. Dinitz, editors, CRC Press (1996), ISBN 0-8493-8948-8, pp. 256-260; see enumeration table on p. 259.

⁹ Curtis P. Brown, Michal Cenk, Richard A Games, Joseph J. Rushanan, and Oscar Moreno, *New Enumeration Results for Costas Arrays*, IEEE Symposium on Information Theory, January 1993, p. 405.

¹⁰ Bill Correll and James K Beard, *Selectioning Appropriate Costas Arrays for Target Detection*, 2017 IEEE Radar Conference, May 8-12, 2017, paper 5351 in Waveform Design 2, *Grand Ballroom 1* (Grand Level)

¹¹ James K Beard, *Costas array generator polynomials in finite fields*, 42nd Annual Conference on Information Sciences and Systems (CISS), March 20, 2008, DOI: 10.1109/CISS.2008.4558709