A faded, grayscale background image of a man with a beard and glasses, wearing a suit and tie. The image is centered and serves as a backdrop for the text.

Physics, Chemistry, and Mathematics of Photography

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Part I of II, February 26, 2003

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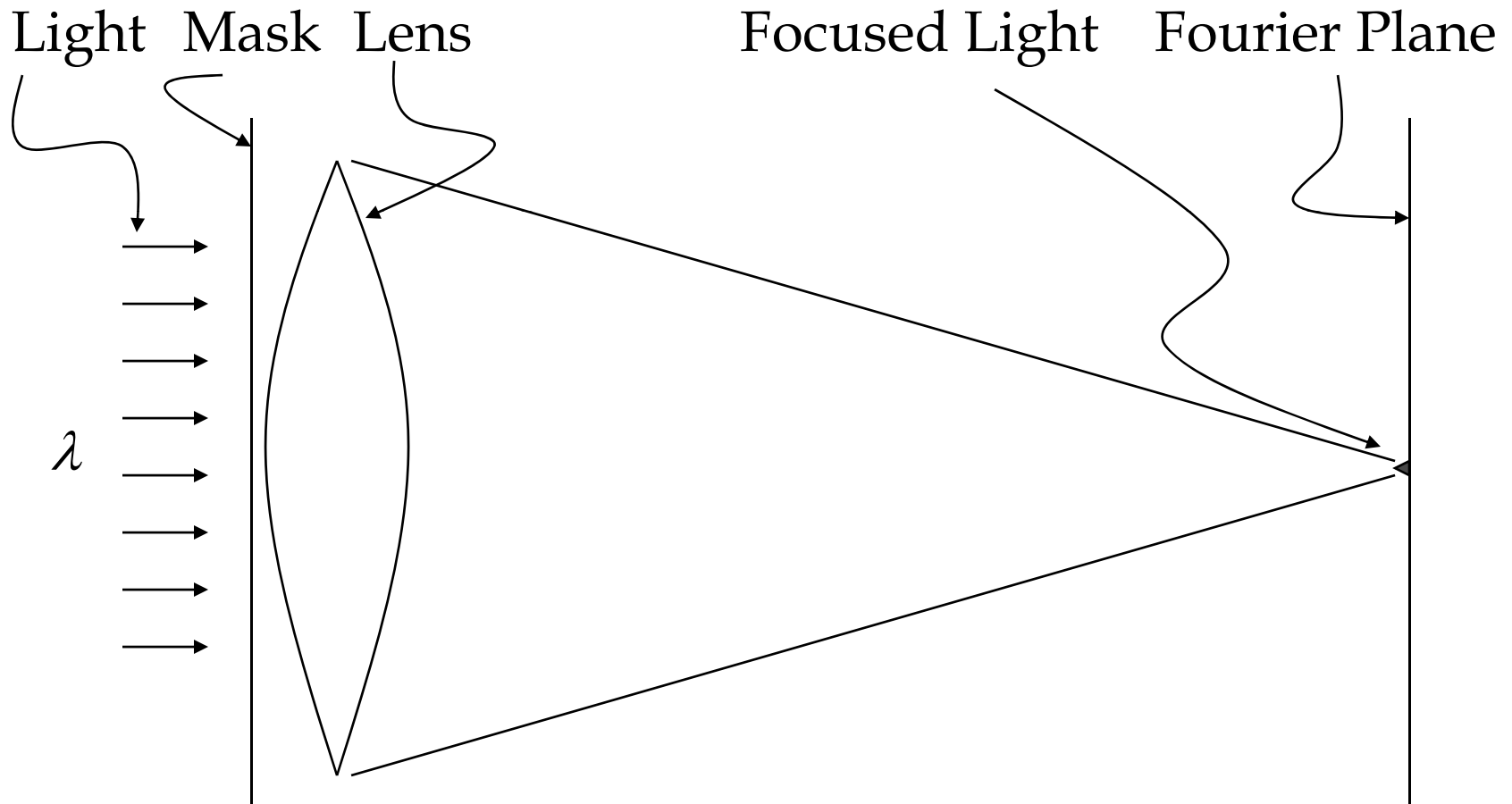
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Topics

- Part I: Resolution
 - On the film
 - On the print
 - On the slides
 - In television
- Part II: Color, shading, and prints
 - Contrast, color, and the Zone System
 - Lens design
 - Digital and conventional photography
 - The portal: scanning and scanners



Optical Fourier Transform



Optical Fourier Transform

- Collimated monochromatic impinging light
- Mask modulates light intensity
- Lens makes all effective path lengths to focal point equal
- Light at focal point is

“Electro-Optical
Systems Analysis,”
K. Seyrafi, p 174-177

$$a(\rho, \varphi) = \int_0^{\frac{D_0}{2}} \int_{-\pi}^{\pi} m(r, \theta) \cdot \exp\left(-j \cdot \frac{2\pi}{\lambda} \cdot r \cdot \rho \cdot \cos(\theta - \varphi)\right) \cdot r \cdot dr \cdot d\theta$$

$r, \theta \leftrightarrow$ Lens Plane; r is distance

$\rho, \varphi \leftrightarrow$ Fourier Plane; ρ is half-cone angle

The Airy Disk

- An open, uniformly weighted circular aperture of diameter D_0
- Intensity on the Fourier plane is

$$a(\rho) = \frac{\pi \cdot D_0^2}{4} \cdot \frac{2 \cdot J_1\left(\frac{\pi \cdot D_0 \cdot \rho}{\lambda}\right)}{\pi \cdot D_0 \cdot \rho}$$

“Electro-Optical
Systems Analysis,”
K. Seyrafi, p 174-177

- Resolution is peak-to-null distance

$$\Delta\rho = \frac{1.22 \cdot \lambda}{D_0}$$

The Diffraction Limit on the Focal Plane

- The f/stop or f-number is

$$(f / no) = \frac{f}{D_0}$$

- The numerical aperture for a lens is

$$NA = n' \cdot \sin(u') = \frac{1}{2 \cdot (f / no)}$$

n' = refractive index
 u' = half-cone angle

- Resolution distance is

$$\Delta\rho \cdot f = 1.22 \cdot (f / no) \cdot \lambda = \frac{0.61 \cdot \lambda}{NA}$$

Lines per Millimeter

- A resolvable line is two resolution elements
 - Two lines must have a resolvable space between them
 - The distance between the line and the resolvable space is a resolvable element
- The number of lines per millimeter is

$$lpmm = \frac{1}{2 \cdot \Delta\rho \cdot f} = \frac{1}{2.44 \cdot (f / no) \cdot \lambda} = \frac{NA}{1.22 \cdot \lambda}$$

Resolution on 35 mm Film

- Film resolution is 80 to 200 lines per mm
- Optics limit
 - Diffraction limit is about 133 lines per mm at f/5.6
 - Achieved in laboratory: 80 to 100 lines per mm
 - Achieved by the photographer: 40-80 lines per mm

Practical limit is 40 – 80 lines per mm

Resolution on 70 mm Film

- Film resolution is 80 to 200 lines per mm
- Optics limit
 - Diffraction limit is about 65 lines per mm at f/11
 - Achieved in laboratory: 45 to 80 lines per mm
 - Achieved by the photographer: 25-50 lines per mm

Practical limit is 25 – 50 lines per mm

Number of Pixels

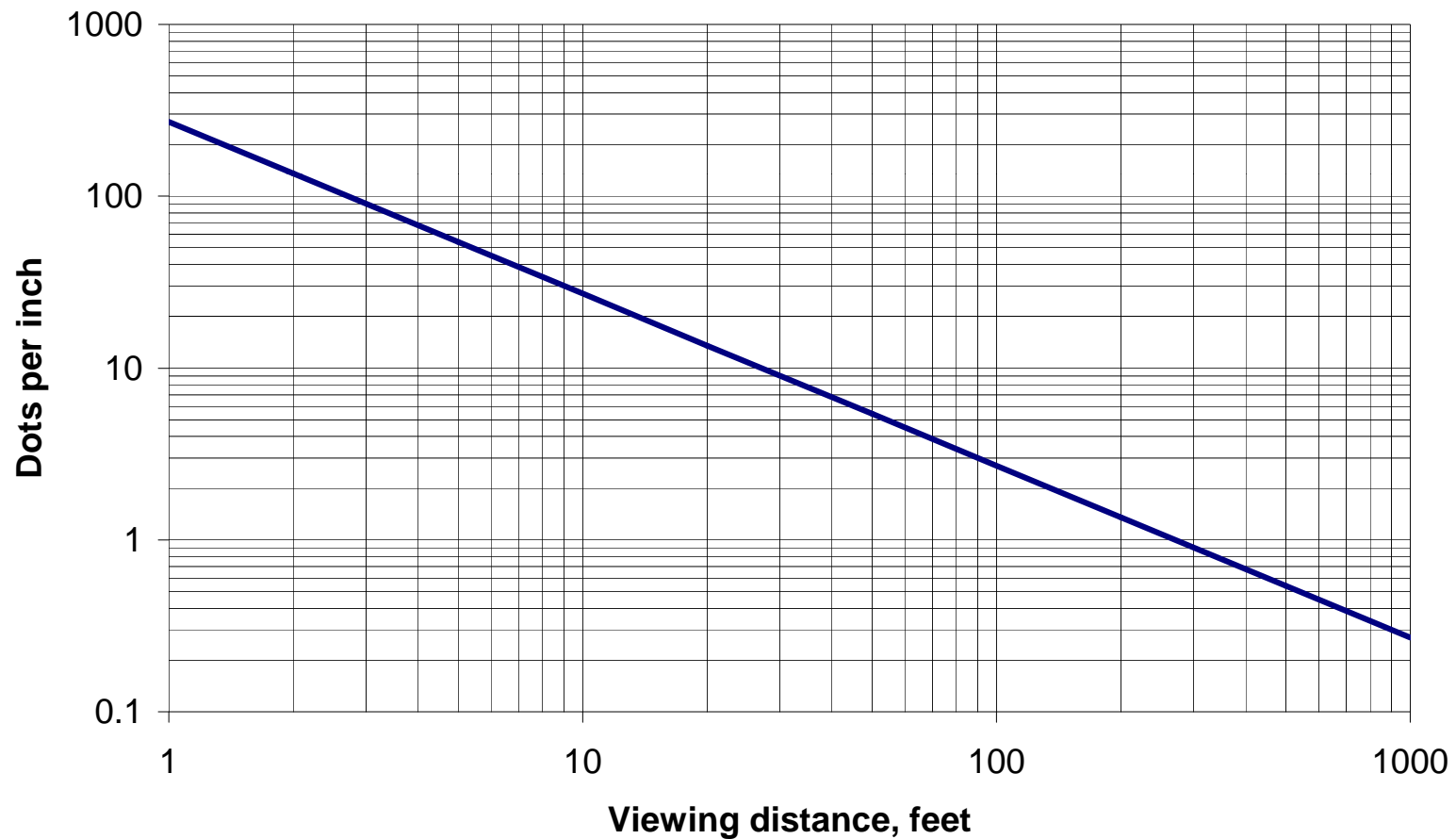
- In 35 mm film
 - Image is 24 mm by 36 mm
 - 4 X 3 aspect limit is 24 mm X 32 mm
 - 4.7 M at 40 lines per mm ($M = 1024^2$ pixels)
 - 18.75 M at 80 lines per mm
- In 70 mm film
 - Image is 6 X 7 cm
 - 22.4 M in 5.25 X 7 cm at 40 lines per mm



Print Resolution

- Eye is about a 24 mm focal length
- F-stop of pupil is $f/2.8$ to $f/32$
 - Typical pupil is $f/11$ in bright light
 - Eye resolution is 0.3 milliradians for green light
- Print resolution
 - At two feet viewing distance
 - About 144 dots per inch

Print Resolution vs. Distance



Scanning Slides and Negatives

- Resolution

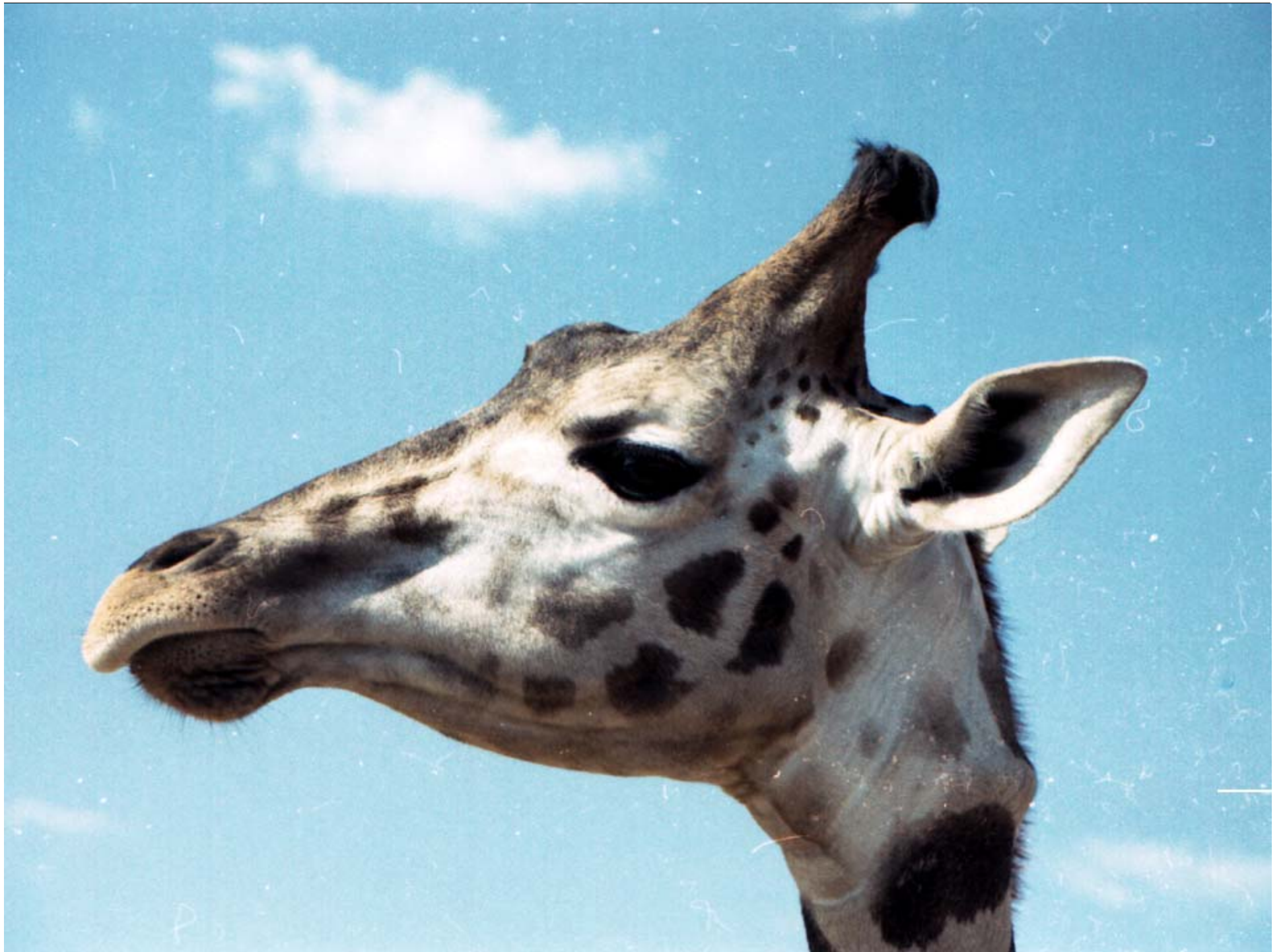
- Scale factor from lines per mm to DPI

$$25.4 \frac{\text{mm}}{\text{in}} \cdot 2 \frac{\text{dots}}{\text{line}} = 50.8 \frac{\text{dots / in}}{\text{line / mm}}$$

- 80 line per mm is 4000 DPI

- Vendors of slide/negative scanners

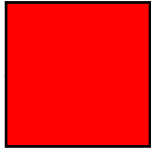
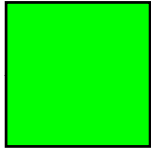
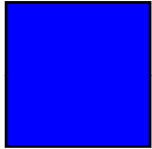
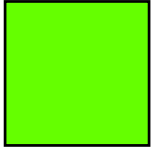
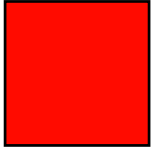
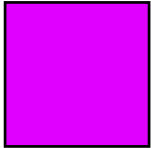
- SmartDisk: 2700 or 3600 DPI
- Nikon Coolscan, Canoscan, Microtek, Polaroid: 4000 DPI
- Minolta Dimage: 2820, 4800 DPI



NTSC Color TV

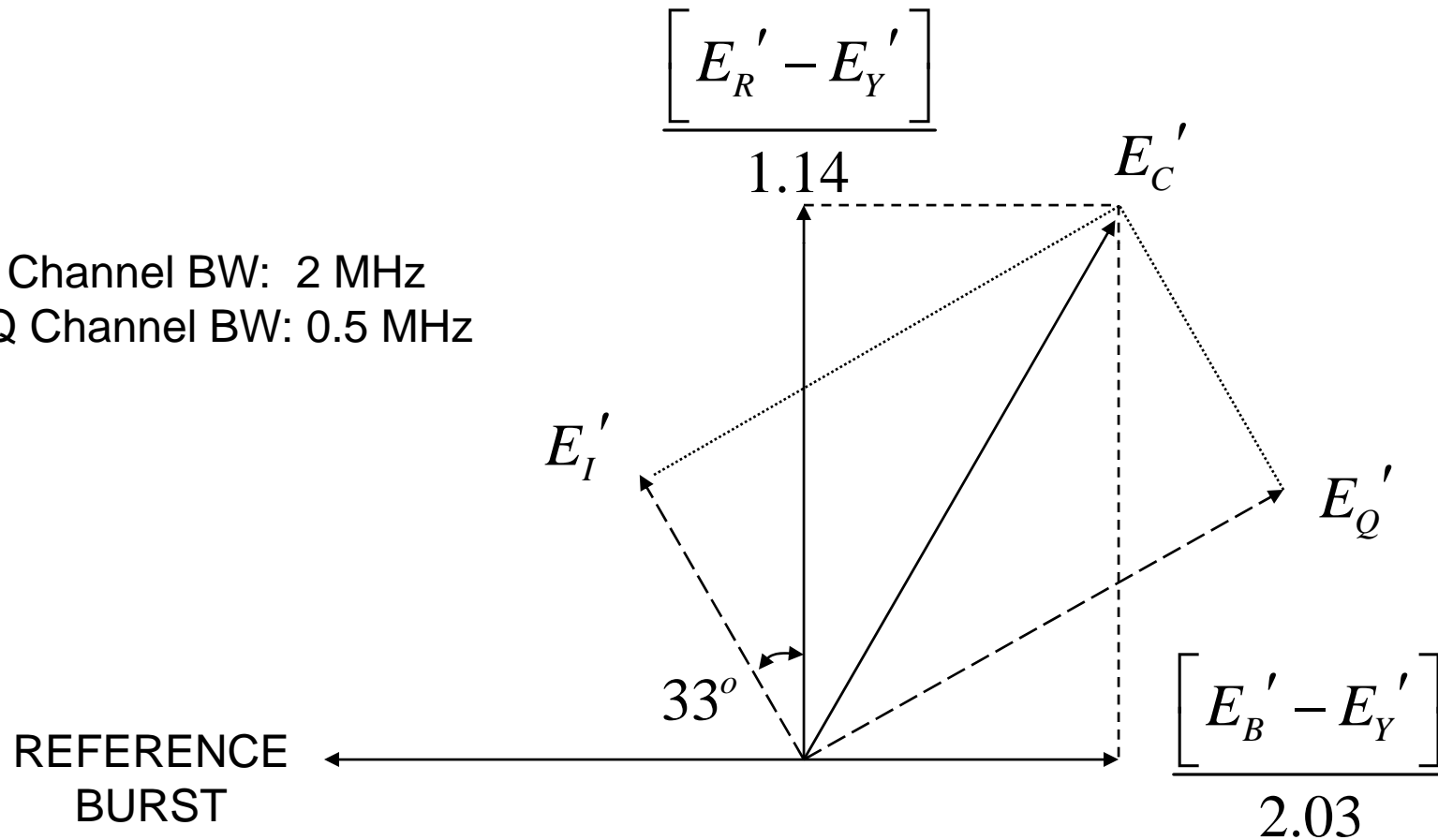
- RGB is encoded for transmission
 - Illuminance, weighted combination
 - I and Q channels to carry color
 - Signal is compatible with pre-color TV
- Resolution
 - Limited by channel bandwidth to 220 lines
 - Color is less; relationship is complex

The Encoding Matrix

| | | E'_R | E'_G | E'_B | | |
|--------|---|---|--|---|-------|---|
| | |  |  |  | | |
| E'_Y |  | [| 0.30 | 0.59 | 0.11 |] |
| E'_I |  | | 0.60 | -0.28 | -0.32 | |
| E'_Q |  | | 0.21 | -0.52 | 0.31 | |

NTSC Color Encoding

I Channel BW: 2 MHz
Q Channel BW: 0.5 MHz



NTSC Summary

- Resolution
 - Limited by TV channel bandwidth
 - About 200 by 480
 - Color is less bandwidth – complex relationship
 - Eye sees 200 by 480
 - Frame averaging with motion enhances perception
- Color
 - Purity and quality are not a problem
 - Blue is lower resolution than red, green

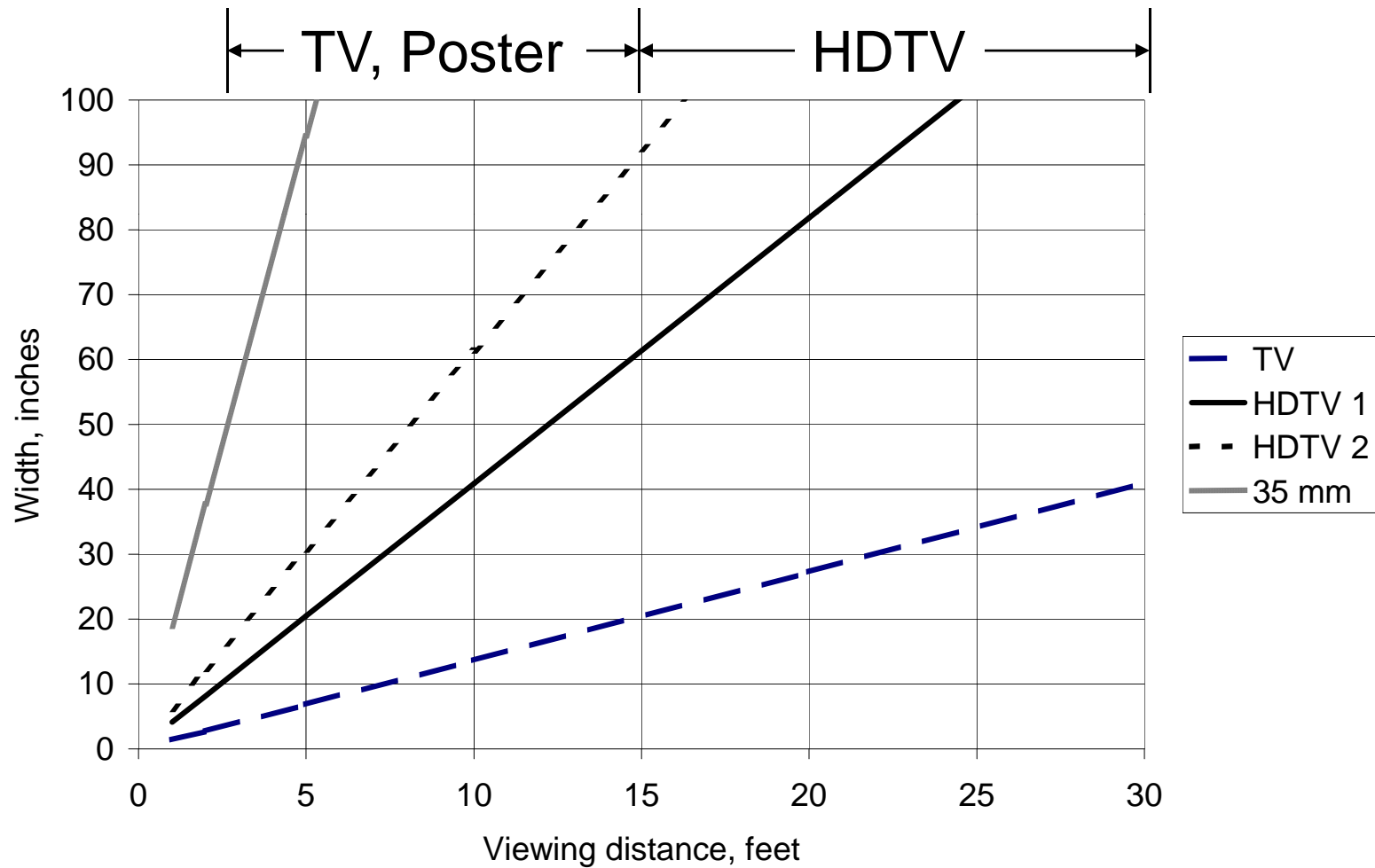
HDTV

- Information given here is from
 - <http://www.ee.washington.edu/conselec/CE/kuhn/hdtv/95x5.htm>
- Aspect ratio remains 16:9
- Resolution – either of
 - 1280 X 720 (1 MPX)
 - 1920 X 1080 (2 MPX)

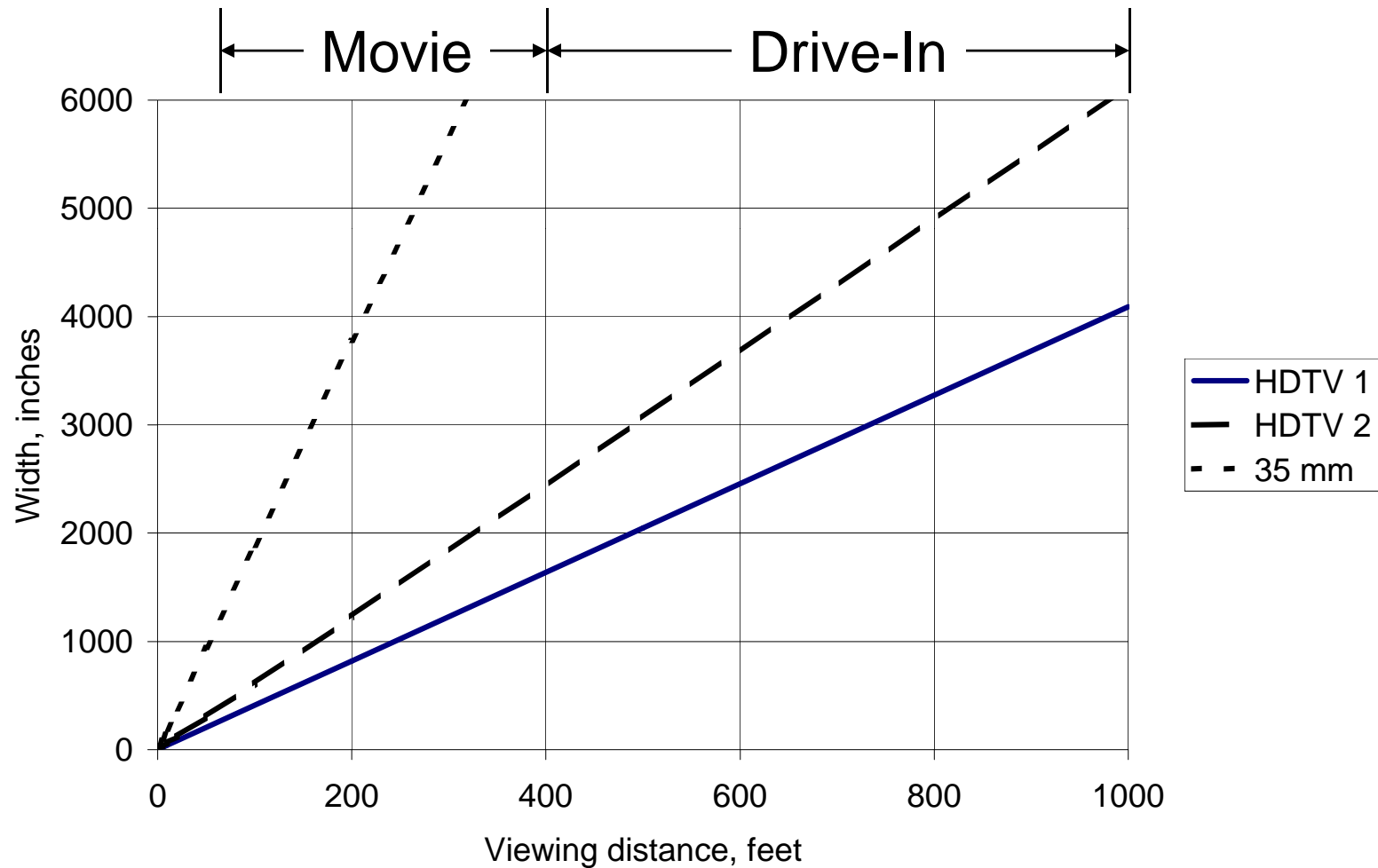
Field of View

| <i>Format</i> | <i>Field of View, Degrees</i> |
|---------------|-------------------------------|
| TV | 6.5 |
| HDTV 1 | 20 |
| HDTV 2 | 30 |
| 35 mm | 104 |

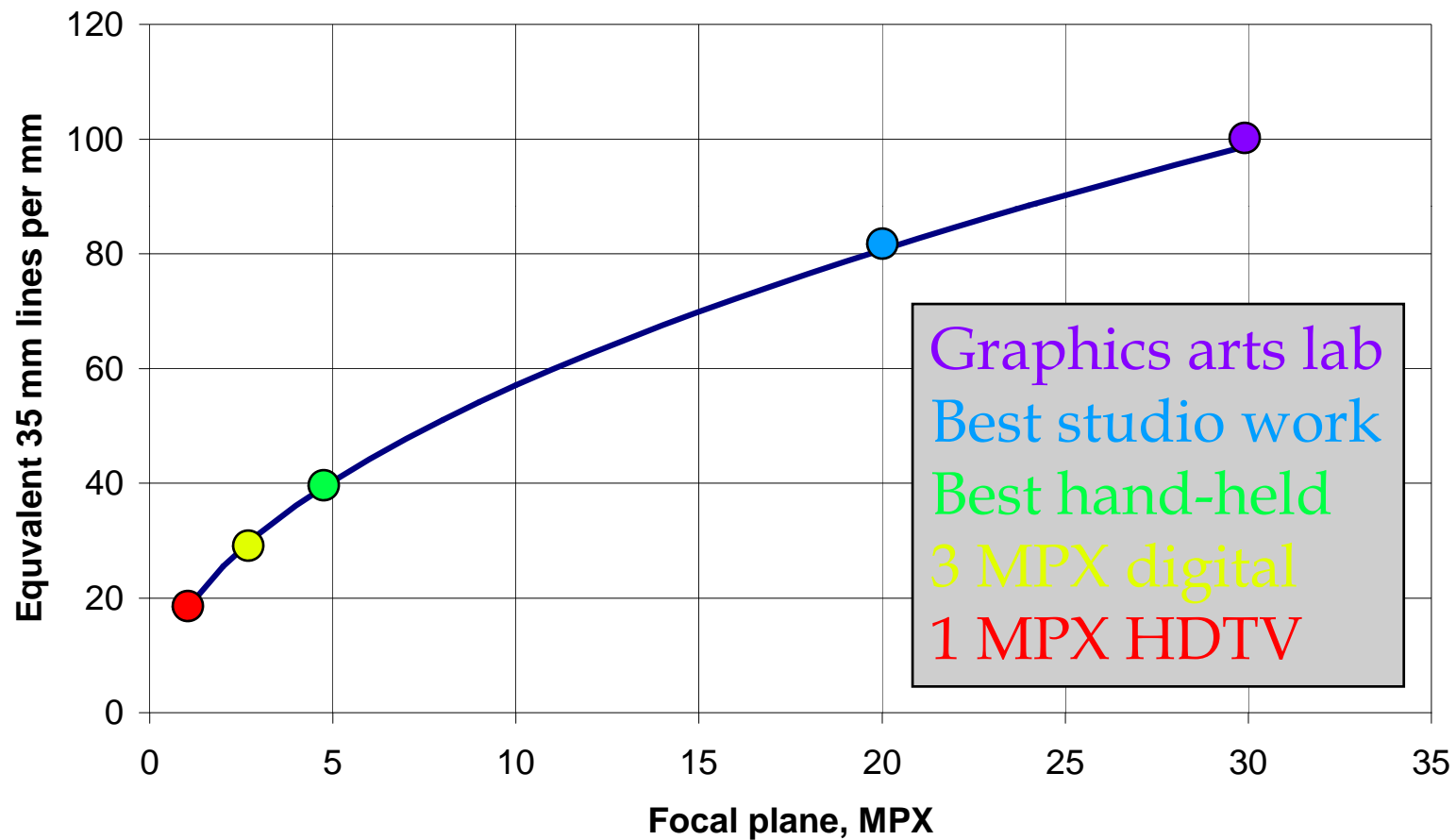
Viewing Image Size vs. Distance



Viewing Image Size vs. Distance



Comparing Digital and 35 mm Focal Plane Resolution



References

- HDTV
 - <http://www.ee.washington.edu/conselec/CE/kuhn/hdtv/95x5.htm>
- Foveon digital photography focal planes
 - <http://www.foveon.com/>
 - <http://www.sigma-photo.com/>
- Popular Photography Magazine
 - <http://www.popphoto.com/>



