

AN EFFICIENT, RELIABLE APPROACH
FOR
HYDROACOUSTICAL BEARINGS ONLY TARGET MOTION ANALYSIS

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ABSTRACT

The hydroacoustical bearings only target motion analysis (TMA) problem is that of determining target location, heading and velocity from target bearings. It must be solved using noisy data. Also, the hydroacoustical bearing trackers used for the measurements produce poor quality data because of the nature of the ocean medium. Since bearings only TMA is used on submarines for fire control, the performance of the TMA subsystems must be accurate, reliable, and computationally efficient. Historically, this problem has been most effectively solved using plotting boards and extended Kalman filters. Recent work has established the performance superiority of batch estimation techniques.

The work presented here is a batch maximum likelihood estimation technique which solves the computational problems associated with batch estimation of the past. In addition, the reliability of the approach given here is appropriate for "hands off" application in a real time subsystem. The problem is solved using state of the art computational and statistical estimation techniques and a coordinate system differing slightly from those used in the past. The formulation of the technique is a response to the hydroacoustical, computational, and mathematical aspects of the TMA problem, within the constraints of performance requirements and real time implementation considerations. Simulation results are presented which show the performance of the maximum likelihood TMA estimator to be superior to a Kalman TMA estimator.

1.1 Historical Summary

Hydroacoustical Target motion analysis (TMA) is implemented using target bearings obtained from a passive sonar at low signal to noise. The function of TMA is to determine target range, heading, and velocity, and to produce reliable estimates of the accuracy of these parameters. Originally, TMA was done using bearing and time marks on plotting boards. A slide rule technique approximating the plotting board solution was introduced about 1950. This technique is known as Ekelund ranging, and is still in wide use today.

Real time application of Kalman filter TMA techniques began about 1970 with the introduction of solid state digital computers in submarines. At about the same time, the technology of application of extended (to nonlinear problems) Kalman filtering techniques matured to the point that useful TMA could be accomplished. The earliest Kalman filters were four-state target motion parameter estimators using Cartesian coordinates (Ref. 1). The most used of these was the Pseudo-Linear Kalman Tracker, introduced and developed by Aidala and others (Refs. 2, 3, 4, and 5). In the late 1970's, IBM first proposed a polar coordinates for target motion parameters in an extended Kalman filter to replace the Pseudo-Linear algorithm (Refs. 6 and 7). This algorithm, called the Modified Polar (MP) Kalman tracker, overcame most of the problems of the algorithms using Cartesian coordinates, but it is not as statistically efficient as some of them. Statistical efficiency is defined classically as the ratio of the best attainable variance of any estimator to that of the estimator used (Refs. 8, 9, and 10).

In TMA, as in most applications, the superior performance of batch techniques over recursive or Kalman techniques is well accepted. A recent paper (Ref. 11) has demonstrated this in principle for the hydroacoustical bearings-only TMA problem. Attempts at batch TMA in the past have been limited to formulations using the sine and cosine of the measured bearing in simple least-squares formulations (Ref. 12); these algorithms are known to be strongly biased to low range estimates in situations where existing algorithms do not fail.

Maximum likelihood estimators are asymptotically efficient, which means that their statistical efficiency approaches 100% as the number of measurements used increases (Refs. 8, 9, and 10). However, prior to this work, exact maximum likelihood estimators have not been applied successfully to the hydroacoustical bearings only TMA problem in real time systems because of mathematical and computational problems. A recent paper gives an approximation of a maximum likelihood estimator which is valid when the total bearing swept by the target over the TMA run is small (Ref. 13). The algorithm presented here is an exact maximum likelihood estimator, valid for any TMA geometry.

1.2 Technical Summary

The problems to be solved in hydroacoustical bearings-only TMA fall into three categories: hydroacoustical, computational, and mathematical. Hydroacoustical problems are problems in the bearing measurements due to wandering raypaths caused by eddies and other hydroacoustical phenomena. In addition, hardware features such as nonGaussian bearing measurement errors and variable integration time in the bearing tracker also cause problems. Computational problems are those of dynamic range versus digital word length and amount of arithmetic required versus available computer resources. Mathematical problems are singularities or stationary points in the models used in the estimation algorithm, such as zero range in the Cartesian coordinate system. In these cases, a zero range estimate will interact with the algorithm in such a manner as to stop the estimates from changing further. This is known as range collapse, and happens sometimes when the amount of data is small and range information is not yet present. Other mathematical problems are caused when the target motion model or the measurement model is singular in one of the state variables for a value near that of the true solution. For example, in nonlinear batch processes, the state vector estimate is found by an iterative process. If the models for the target motion or the bearing measurement process have singularities near the true value of the state vector, this recursive process may be difficult to implement reliably. In addition, singularities near the solution cause increased numbers of iterations to be required in the iterative process, which can cause excessive computational requirements. Another class of mathematical problems concerns statistical efficiency. The statistical efficiency of the estimate is known to decrease with the nonlinearity of the estimation equation, particularly when the number of measurements is small (Ref. 10). Thus, highest statistical efficiency is obtained when the state vector is a near-linear function of the measurements, or when a batch estimator is used.

Hydroacoustical problems are solved by allowing a block scale factor to be applied to measurement errors as a scale factor. This allows bearing wander due to hydroacoustical effects to be modelled as measurement error. Correlations between bearing errors are neglected by a batch estimator because trends over the entire data base convey much greater measurability of the state vector. Variable integration time of the bearing tracker causes corresponding correlation in the bearing measurement errors; these correlations are neglected by a batch estimator by the same mechanism as similarly correlated errors caused by hydroacoustical effects.

Computational problems are solved by posing each iteration of the batch estimation problem as an overdetermined least squares problem and solving it with Householder transformations (Refs. 14, 15). This allows computation in single precision floating point arithmetic using a 24 bit mantissa, which reduces computational requirements. In order to minimize propagation of numerical errors, the number and order of the states is adaptive. That is, states which do not meet a threshold of observability are dropped out of the computation and a nominal value is used in the measurement models, and the states with the highest degree of observability are estimated first. These provisions prevent noisy state estimates from causing propagation of errors into observable states.

Mathematical problems are solved by the selection of states. The states are chosen so that the algebraic functions in the system dynamics and measurement models, considered as functions of the state variables, have no singular points near the best estimate of the state variables. They are also chosen so that their estimates are not correlated in an extreme degree, which prevents numerical problems due to a poorly conditioned (nearly singular) covariance matrix. Statistical efficiency is assured by the use of all available bearing measurements in a single maximum likelihood estimator.

The statistical and mathematical notation of a maximum likelihood estimator for hydroacoustical bearings only TMA is presented first. Then, the selection of the state vector which helps to solve the mathematical problems is given. Finally, implementation considerations are discussed.

2.1 Maximum Likelihood TMA

At a particular instant in time, the hydroacoustical bearings only TMA problem can be stated as follows: given a data base of target bearings and own-ship positions, to estimate the current target range, course, and speed. Thus, the problem can be stated as a classical parameter estimation problem (Refs. 8, 9, 10, 14, 15, and 16). This is the case when a Kalman filter is used with a system dynamics model containing no plant noise, or where plant noise is included only to prevent covariance collapse (Refs. 8 and 16). The bearings only TMA problem is stated as a maximum likelihood problem as follows: given the bearing measurements \underline{y} (in vector format), to find the state vector \underline{x} which maximized the conditional probability density function $p(\underline{y}|\underline{x})$. The target position, heading and velocity are found from the components of the state vector \underline{x} . The own-ship positions are considered to be part of the model. This process is simplified by using the log likelihood function,

$$L = \ln\left(p(\underline{y}|\underline{x})\right) \quad (1)$$

instead. This is in part because Gaussian and many other probability density functions are simpler in this format. Since the logarithm function is analytic and monotonic, there is a one-to-one correspondence between maxima of $p(\underline{y}|\underline{x})$ and maxima of L . If the bearings are Gaussian (nonGaussian bearings are considered below), then the log likelihood function is

$$L = -0.5 \cdot \left(\underline{y} - \underline{h}(\underline{x})\right)^T \cdot R^{-1} \cdot \left(\underline{y} - \underline{h}(\underline{x})\right) + \text{constant}, \quad (2)$$

where R is the covariance matrix of the bearing measurements and $\underline{h}(\underline{x})$ is the expected bearing as computed from the state vector \underline{x} . The time variable is considered to be implied by the index of the component of the \underline{y} and \underline{h} vectors, and the own ship position (and velocity) information is contained in the vector function $\underline{h}(\underline{x})$. The solution to the likelihood equation

$$\frac{\partial L}{\partial \underline{x}} = 0 \quad (3)$$

is found by conventional techniques. Before proceeding, however, it is expedient to make some variable changes. Since R is nearly always diagonal or tridiagonal, its Cholesky factor (Ref. 14) can be found and inverted with essentially no computational effort. Using the notation

$$R^{-1} = Y^T \cdot Y \quad (4)$$

for the inverse of the Cholesky factor (Ref. 14) Y , the bearing measurement and estimation vectors \underline{y} and $\underline{h}(\underline{x})$ can be normalized by the bearing variances to form the normalized vectors

$$\underline{z} = Y \cdot \underline{y}, \underline{a}(\underline{x}) = Y \cdot \underline{h}(\underline{x}). \quad (5)$$

This allows the log likelihood function to be written as

$$L = -0.5 \cdot (\underline{z} - \underline{a}(\underline{x}))^T \cdot (\underline{z} - \underline{a}(\underline{x})) + \text{constant} \quad (6)$$

and the likelihood equation as

$$\frac{\partial L}{\partial \underline{x}} = A^T \cdot (\underline{z} - \underline{a}(\underline{x})) = \underline{0}, \quad A = \frac{\partial \underline{a}(\underline{x})}{\partial \underline{x}} = Y \cdot \frac{\partial \underline{h}(\underline{x})}{\partial \underline{x}}. \quad (7)$$

Furthermore, the Fisher information matrix is defined as the negative of the Hessian matrix, or second gradient, of the log likelihood function,

$$V = -\frac{\partial^2 L}{\partial \underline{x}^2}. \quad (8)$$

The inverse of this matrix is, to the first order, the covariance matrix of the error in the maximum likelihood estimate of the state vector.

If the likelihood equation, Eq. (7) were linear, the maximum likelihood estimate could be found by a simple matrix inversion, and the resulting estimate would be statistically efficient; that is, its statistical efficiency would be 100%. In any case, Fisher and others have shown (Ref. 10) that maximum likelihood estimators using a large number of measurements have the property that the localization ellipsoid attained by a maximum likelihood estimator is contained wholly within the localization ellipsoid attained by any estimator. The localization ellipsoid is defined as the surface defined by setting the probability density function of the estimate to a particular value, and is most meaningful when the probability density of the estimator is Gaussian. If the estimate is Gaussian, the one-sigma limit is the most often chosen value for defining a localization ellipsoid for the purposes of comparing estimators. The localization ellipsoid has the advantage of taking into account the entire covariance matrix of the error in an estimator, not just the variances of the estimates of each state. Furthermore, the maximum likelihood estimator is always asymptotically efficient, which means that the statistical efficiency increases monotonically with an upper bound of 100% as the number of measurements is increased. Finally, the errors in a maximum likelihood estimator are Gaussian in the limit as the number of measurements increases, with, in the limit, a mean of zero and a covariance of the inverse of the Fisher information matrix (see Eq. (8)).

Since the likelihood equation is, in general, nonlinear, its solution is usually found iteratively. An error vector \underline{e} and its derivative are defined by

$$\underline{e} = \frac{\partial \mathcal{L}}{\partial \underline{x}}, \quad \frac{\partial \underline{e}}{\partial \underline{x}} = -V; \quad (9)$$

a vector extension of Newton's method, or steepest descent, gives the iteration required to drive \underline{e} to zero and thus solve Eq. (7) as

$$\hat{\underline{x}} = \hat{\underline{x}}(-) + V^{-1} \cdot A^T \cdot (\underline{z} - \underline{a}(\hat{\underline{x}}(-))). \quad (10)$$

If the higher order derivatives of $\underline{a}(\underline{x})$ are neglected, then V can be approximated by

$$V \cong A^T \cdot A. \quad (11)$$

This approximation results in an algebraic problem statement identical to that obtained by Bayesian methods. Note that an initial value for $\hat{\underline{x}}(-)$ must be assumed, and that convergence of the iterative process and other practical issues have not been addressed at this point.

NonGaussian measurement statistics can be considered by extending the definition of the probability density function of the bearing measurements to an Edgeworth series. The skewness and kurtosis of the bearing measurements can be added as two extra states and the corresponding Edgeworth terms added into the likelihood equation and the definition of the Fisher information matrix. These terms are sufficient to ensure asymptotic efficiency of the estimator whenever the Edgeworth series converges (Refs 10, 17 and 18).

2.2 The State Vector

The state vector is expressed in the log polar coordinate system. The origin is the phase center of the bearing sensor. The notation of the log polar coordinate system is developed from the Cartesian system. If the relative target positions in the East and North directions are denoted by X_c and Y_c (the subscripts are used to avoid conflicts in notation), then range R , bearing b , and a complex variable w are defined by

$$w = Y_c + j \cdot X_c = R \cdot \exp(-j \cdot b). \quad (12)$$

The log polar coordinates of target position, r and b , are defined as the real and imaginary components of the natural logarithm of w ,

$$r + j \cdot b = \ln\left(\frac{w}{R_c}\right) = \ln\left(\frac{R}{R_c}\right) + j \cdot \tan^{-1}\left(\frac{X}{Y}\right) \quad (13)$$

where R_c is a free parameter chosen to adjust the scaling of r . A convenient value of R_c is the range used to initialize Eq. (10); the initial value of r is then zero. The state vector is completed by the derivatives of r and b with respect to time,

$$r' + j \cdot b' = \frac{w'}{w}. \quad (14)$$

The time variable is taken as zero at current time. The components of the state vector are r_0 , b_0 , r'_0 and b'_0 , where the subscript 0 denotes values at current time.

The complex variable w can be expressed as complex target position minus own ship position,

$$w = w_t - w_{os}. \quad (15)$$

This allows combining own ship position and velocity data and a nonaccelerating target motion model to be used to find a general state variable extrapolation equation. This is done by combining Eqs. (13) and (15),

$$R_c \cdot \exp(r + j \cdot b) = w_t - w_{os} \quad (16)$$

and expressing the result and its first derivative at current time, to form

$$R_c \cdot \exp(r_0 + j \cdot b_0) = w_{t0} - w_{os0} \quad (17)$$

and

$$(r'_0 + j \cdot b'_0) \cdot R_c \cdot \exp(r_0 + j \cdot b_0) = w'_t - w'_{os0}. \quad (18)$$

For a nonaccelerating target,

$$w_t = w_{t0} + w'_t \cdot t, \quad w'_t = \text{constant}. \quad (19)$$

By solving Eqs. (17) and (18) for w_{t0} and w'_t , the target position w_t can be found as a function of the state variables at current time. Substituting the result into Eq. (16) gives the general expression

$$R_c \cdot \exp(r + j \cdot b) = (1 + (r'_0 + j \cdot b'_0) \cdot t) \cdot R_c \cdot \exp(r_0 + j \cdot b_0) + dw_{os} \quad (20)$$

where

$$dw_{os} = w_{os0} + w'_{os0} \cdot t - w_{os} \quad (21)$$

is the complex distance over which the own ship has accelerated by maneuvering between time t and current time. The natural logarithm of Eq. (20) and its time derivative can be used to find the state variables at any time t from the state variables at current time, and the extrapolated bearings thus represent the function $\underline{h}(\underline{x})$ in Eqs. (2) and (5). These equations are

$$r + j \cdot b = r_0 + j \cdot b_0 + \ln \left[1 + (r'_0 + j \cdot b'_0) \cdot t + \frac{dw_{os}}{R_0} \cdot \exp(-j \cdot b_0) \right] \quad (22)$$

and

$$r' + j \cdot b' = \frac{r'_0 + j \cdot b'_0 + \frac{w'_{os0} - w'_{os}}{R_0} \cdot \exp(-j \cdot b_0)}{1 + (r'_0 + j \cdot b'_0) \cdot t + \frac{dw_{os}}{R_0} \cdot \exp(-j \cdot b_0)} \quad (23)$$

Each bearing measurement time t_i results in another element of the vector $\underline{h}(\underline{x})$. The function $\underline{a}(\underline{x})$ is found by dividing each element of $\underline{h}(\underline{x})$ by the standard deviation of the bearing measurement taken at the corresponding time t_i .

The partial derivatives which make up the matrix A matrix are found by taking the partial derivative of both sides of the imaginary part of Eq. (22) with respect to the state vector. This results in the partial derivative of the bearing measurement with respect to the state vector. Since Eq. (22) is the complex variable $r + j \cdot b$ as an analytic function of the complex variables $r_0 + j \cdot b_0$ and $r'_0 + j \cdot b'_0$, the Cauchy-Riemann equations (Ref. 18) can be used to simplify the derivation. The result is

$$\left. \begin{aligned} \frac{\partial b}{\partial b_0} + j \cdot \frac{\partial b}{\partial r_0} &= \frac{1 + (r'_0 + j \cdot b'_0) \cdot t}{1 + (r'_0 + j \cdot b'_0) \cdot t + \frac{dw_{os}}{R_0} \cdot \exp(-j \cdot b_0)} \\ \frac{\partial b}{\partial b'_0} + j \cdot \frac{\partial b}{\partial r'_0} &= \frac{t}{1 + (r'_0 + j \cdot b'_0) \cdot t + \frac{dw_{os}}{R_0} \cdot \exp(-j \cdot b_0)} \end{aligned} \right\} \quad (24)$$

The log polar coordinate system is very similar to the MP coordinate system referred to earlier. The difference is the use of $\ln(R/R_c)$ for the range state variable where the MP uses $1/R$ for that state. The partial derivatives are the same, except that those corresponding to the range state are scaled by a factor of $-1/R^2$. The difference is in the implementation of the steepest descent iteration, Eq. (10). In any practical case, corrections to the range state in the $1/R$ domain will often cause the updated state variable to fall near the singular point of $1/R=0$, which in turn will drastically affect the next iteration. In addition, any nonlinear iterative process must have **ad hoc** limits on the state variable correction to prevent numerical problems. These limits are best placed remotely enough so that the iterative process described by Eq. (10) follows a locus to the final solution freely. This is not possible for the MP because a singularity is always very near the state variable

estimate in the 1/R domain. A similar problem occurs when Cartesian coordinates are used, because corrections to the position states often cause a small range estimate to occur when range observability is poor. In Cartesian coordinates, the V^{-1} matrix becomes very small when range is small, so that subsequent iterations will not cause the range estimate to increase rapidly to its proper value.

When recursive (Kalman) estimators are used in hydroacoustical bearings only TMA, Eq. (10) is replaced by the familiar Kalman state update equation, and the same problems occur. However, there is one very important difference. When only one measurement is processed at a time, the change in bearing is small; the update of the 1/R state is proportional, in the first order, to the change in bearing. Since only one measurement is present, the principle of the most nearly linear estimator yielding the highest statistical efficiency is operative. As a result, the MP coordinate system performs slightly better than the log polar in recursive estimators. However, neither the log polar nor the MP Kalman filter has the statistical efficiency of Kalman filters using Cartesian coordinates, because the state vector updates in these filters are more nearly linear in the measurement residuals than the MP. However, Kalman filters using Cartesian coordinates suffer from poorly conditioned covariance matrices due to highly eccentric localization ellipses in addition to their range collapse problems. The overall result is that the MP is the best overall compromise between mathematical and numerical problems and statistical efficiency. In batch processing, statistical efficiency is not a strong function of the coordinate system and tractability of Eq. (10) is the controlling factor.

Prior to the point where the algorithm is simply tracking, the initial state vector estimate used in Eq. (10) is found using four bearings spaced evenly through the existing data base. Complex variables summed around a rectangle defined by the own ship and target at current time and at time t_i yield the equation

$$R_0 \cdot \exp(j \cdot b_0) + w'_t \cdot dt_i - R_i \cdot \exp(j \cdot b_i) - (w_{osi} - w_{os0}) = 0 \quad (25)$$

where w'_t is the target velocity, expressed as a complex variable, and w_{osi} is the own ship position at time t_i . This is the plotting board triangulation ranging geometry equation. Placing the unknowns on the left side of the equals sign results in the form

$$R_0 \cdot \exp(j \cdot b_0) + w'_t \cdot dt_i - R_i \cdot \exp(j \cdot b_i) = w_{osi} - w_{os0} \cdot \quad (26)$$

The unknown R_i can be eliminated by multiplying both sides of Eq. (26) by $\exp(-j \cdot b_i)$ and then taking the imaginary part. The result is

$$R_0 \cdot \sin(b_0 - b_i) + \text{Im}\{w'_t \cdot \exp(-j \cdot b_i)\} \cdot dt_i = \text{Im}\{(w_{osi} - w_{os0}) \cdot \exp(-j \cdot b_i)\} \cdot \quad (27)$$

This is a real linear inhomogeneous equation in three unknowns: R_0 and the real and imaginary parts of w'_t . The current bearing plus three old bearings adds up to three linear

inhomogeneous equations in three unknowns. Given w'_t , r'_0 and b'_0 can be found from Eq. (18). The initial state vector is then completely determined from the four bearings.

Equation (27) can be reduced further by considering $R_0 \cdot \cos(b_0)$ and $R_0 \cdot \sin(b_0)$ as unknowns along with the real and imaginary parts of w'_i , and the data history presented as an array of such equations. The result is a linear overdetermined least squares problem whose solution is the TMA parameters in Cartesian coordinates. Bearing measurement noise can be accounted for by using standard variance propagation techniques to model the measurement noise as a white Gaussian vector added to the inhomogeneity, allowing weighted least squares to be used. This technique can be further developed by adding data editing, etc. However, since the bearing measurement noise is coupled nonlinearly into the equation in a manner dependent on the state variables, including range, the method tends to develop large biases and break down at low signal to noise (Ref. 12). However, since the maximum likelihood iteration of Eq. (10) is not sensitive to the initial state estimate, this simple four-bearing method suffices for initialization, even for low signal to noise ratios.

As a final note, the Ekelund ranging problem can also be solved exactly using Eq. (27) by noting that the bearing rate is

$$b'_i = \frac{\text{Im}\{(w'_t - w'_{osi}) \cdot \exp(-j \cdot b_i)\}}{R_i} . \quad (28)$$

The Ekelund ranging problem is that of determining range from two observations, one before and one after an own ship maneuver, where bearing and bearing rate are measured. Therefore, substituting Eq. (28) into Eq. (27) yields one equation in R_0 and R_i . Reversing the subscripts 0 and i yields a second linear equation in the ranges at the two observation times. The target velocity can then be found using the ranges thus found in Eq. (26), and the range and bearing at any time can be found by extrapolation. The resulting equations are

$$\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = B^{-1} \cdot \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} \quad (29)$$

where B is a matrix whose elements are

$$B = \begin{bmatrix} b'_1 \cdot dt & \sin(db) \\ \sin(db) & b'_2 \cdot dt \end{bmatrix} \quad (30)$$

where

$$dt = t_2 - t_1, \quad db = b_2 - b_1 \quad (31)$$

and P_1 and P_2 are crossrange projections of own-ship track between measurements,

$$\left. \begin{aligned} P_1 &= \text{Im}\{(w_{os2} - w_{os1} - w'_{os1} \cdot dt) \cdot \exp(-j \cdot b_1)\} \\ P_2 &= \text{Im}\{(w_{os2} - w'_{os2} \cdot dt - w_{os1}) \cdot \exp(-j \cdot b_2)\} \end{aligned} \right\} \quad (32)$$

The estimated target velocity is

$$w'_i = \frac{R_2 \cdot \exp(j \cdot b_2) - R_1 \cdot \exp(j \cdot b_1) + w_{os1} - w_{os2}}{dt} \quad (33)$$

and the interpolated range at arbitrary time t is

$$R(t) \cdot \exp(j \cdot b(t)) = w_{os1} - w_{os}(t) + R_1 \cdot \exp(j \cdot b_1) + w'_i \cdot (t - t_1). \quad (34)$$

It has been found that, if the signal to noise ratio is high and the mean time of the measurements is used as the effective time of the estimated range, the Ekelund range thus obtained is quite accurate and reliable. This "exact" Ekelund range equation is a distinct improvement over approximations to the Ekelund ranging equation which are known to be unreliable when automated.

2.3 Implementation

For all TMA methods, the bearings obtained by the bearing tracker were block averaged for 10 to 20 seconds. This has the benefits of reducing the data rate and yielding a measurement error distribution more nearly Gaussian than the raw single bearings. Also, the integration time of the bearing tracker is not a factor when the block averager time exceeds the tracker integration time.

Although the estimator presented here is inherently a batch processor, it is implemented as if it were a Kalman filter. The data base, no matter how small (or how large) is submitted to the algorithm for estimation of the TMA parameters. However, once observability of the target TMA parameters is obtained, simple target tracking is required. Under these conditions, the state vector from the last estimated target position is extrapolated to current time using Eq. (22) and its time derivative. This condition assures that only one iteration of Eq. (10) is required to update the TMA parameters.

For the purposes of output or display, the variances of the TMA parameters in the desired coordinate frame are found using the standard linearized covariance propagation equation (Ref. 16), as is done when Kalman TMA estimators are used.

There are two important computational techniques used in the implementation: Householder transformations are used to solve the least squares problem posed by Eq. (10) (Refs. 14 and 15) which allows the algorithm to operate successfully in single precision

(a 24 bit mantissa), and the order and number of states estimated at any time is adaptive as discussed earlier. The threshold of observability is that the variance of the estimate must be less than about 10 times the estimate of that particular state. If this condition is not met, that state is dropped out of the estimation procedure and a nominal value used instead. This only happens when the data is insufficient to form an estimate for TMA purposes, but operation of the algorithm is required to monitor observability as the data base accumulates.

There are four significant structural features to the implementation: the choice of the initial state vector for Eq. (10) as previously discussed, the use of the approximation of Eq. (11), **ad hoc** techniques used to control the convergence of the iterative procedure, and data editing techniques.

The approximation of Eq. (11) is, as stated previously, to neglect the third order derivatives of $\underline{h}(\underline{x})$ with respect to the state variables. This can be corrected by recomputing V using the complete expressions after the iterative application of Eq. (10) has converged. Omission of the third derivative terms results in errors in the estimate of the covariance matrix. The rate of convergence obtained with Eq. 10 indicates that these errors are small.

There are three **ad hoc** techniques to control excursion of the state vector during the iterative process of Eq. (10): the length of the state variable correction (with the velocity states multiplied by the time since the last update) is limited to 0.7, the range estimate is bounded below by 10 meters and above by 1000000 meters to avoid arithmetic overflow, and data editing is performed. It was found that the range limits do not significantly affect the rate of convergence, so long as they are at least an order of magnitude away from the true value of range. The state variable correction limit of 0.7 is not critical, but is near an optimum for rate of convergence; this value is near $\ln(2.0)$, which is reminiscent of the "Jarvis fix" (Ref. 1) in which range was not allowed to vary more than a factor of 2 for any single update in the early Kalman TMA estimators using Cartesian coordinates. Data editing is performed only after the iteration of Eq. (10) has converged. At that point, bearing error residuals exceeding 2.75 standard deviations are discarded. Again, this value is not critical; the value chosen would result in about one Gaussian bearing per hour being discarded. An important feature of the data editing is that adaptive block scale factor estimation of the bearing variance is begun after convergence of Eq. (10) is first obtained, which, for real data, allows a much smaller value to be taken for the standard deviation of the measurement. This means that a minimum of three iterations of Eq. (10) are required for any update. It is obvious that this is a very conservative implementation, and that a single pass would suffice once a target track was initiated. The worst case for convergence for a data base allowing observability of all states and initializing Eq. (10) using Eqs. (27) and (18) with four bearing measurements to determine the initial state vector estimate, is 4 iterations of Eq. (10) to obtain convergence, one to estimate a block scale factor for bearing errors, and one for final data editing, for a total of 6 iterations. Very early in a TMA run using real data, lack of observability of the range state combined with inconsistencies in the data due to loss of bearing lock or other hardware problems can occasionally cause the algorithm to hit a software limit of 10 iterations; when this occurs,

the output is simply ignored until more data is available. No computational problems are caused because this never happens unless the data base is very small.

Although the implementation of the maximum likelihood algorithm present here was developed using a great deal of real and synthetic data, the basic properties of the algorithm are demonstrated here using a few synthetic data runs. This is partly for brevity, but there is the additional advantage of being able to compare algorithms for the same run but with different measurement noise variances. The target and own ship geometries are given in Figure 1. The own ship follows a zig zag track, typical of TMA runs, but the target crosses the bow of the own ship. Good observability is obtained, but one own ship leg adds no bearing rate information and the target bearing rate is uniformly extremely high. Figures 2 and 3 show that the range performances of both a log polar extended Kalman filter (EKF) and the batch (maximum likelihood) estimators to be excellent--when the bearing measurement variance is 0.5 degrees. Figures 4 and 5 show that for a bearing measurement variance of four degrees, the EKF is showing the effect of coupling state vector estimate errors into the Kalman gain and covariance estimates, as the EKF shows a large random bias in excess of the variance of the estimate. The batch estimator still performs well for a bearing variance of 8.0 degrees, as shown in Figure 6. The computational loading of the maximum likelihood TMA estimator is about 3.5 times the loading of the Kalman filter, even though both algorithms are used to estimate the TMA parameters for every data point. The source language is RATFOR, the computer used is a Tandy 1000A, and single precision is used for all computations.

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For Figures, please see 23rd IEEE CDC, December 14 1984, Transactions pp. 1255-1256

Figure 1
TMA Geometry

Figure 2
Range Error versus Time (EKF)

Figure 3
Range Error versus Time (Batch)

Figure 4
Range Error versus Time (EKF)

Figure 5
Range Error versus Time (Batch)

Figure 6
Range Error versus Time (Batch)