Maritime Situational Awareness

An Emerging Homeland Defense Capability

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Topics for Today

- Topic 1: Problem definition, requirements, and infrastructure
- Topic 2: Estimation theory: The enabling technology
- Topic 3: Data fusion from multiple sensors
- Topic 4: The C4ISR Architecture

Topic 1: Problem Definition, Requirements, and Infrastructure

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Topic 1: Requirements and Infrastructure

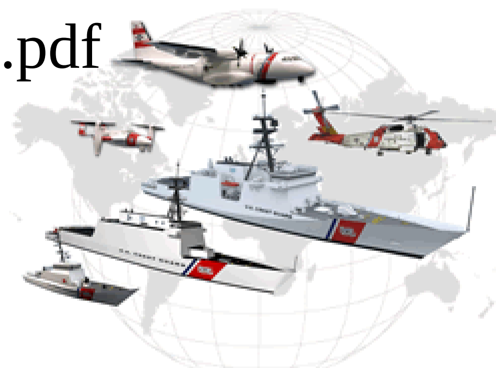
- Mission and requirements
  - USCG Deepwater is the context for some examples
  - The system engineering process is the method
- Infrastructure
  - DII COE is the basis

Defining the Problem and Requirements

- Leverage the system engineering process
- Define the need – the future, emerging, or existing situation that demands a new capability
- Define the mission – the ways that the new capabilities will be defined, deployed, and executed
- Define the requirements – the defining characteristics of the new capabilities and their components

Example: USCG Deepwater

- Main page on USCG site
  - http://www.uscg.mil/deepwater/
- Mission Analysis Statement (MAR, 1995)
  - http://www.uscg.mil/deepwater/pdf/MAR.pdf
- Mission Needs Statement (MNS, 1996)
  - http://www.uscg.mil/deepwater/pdf/mns.pdf
- C4ISR is implicit in future concept
  - System-of-systems approach
  - Integrated with network centric warfare



USCG as Our Example

- MAR, MNS
  - Defined traditional USCG current and emerging threats
  - Defined shortfalls of existing USCG assets
  - Recommended 21st Century capabilities
- Legacy missions merge seamlessly into 2015 missions
  - 1995: Drug interdiction, law enforcement
  - 2005 forward: Seamless homeland security operations
- Deepwater focus in 2005
  - Seamless integration with Navy, other DoD, civilian assets
  - C4ISR is the enabling technology
  - System-of-systems (SoS) overall concept
  - FORCEnet-resident network-centric warfare capability

System-of-Systems FAQ

Why is the Coast Guard using a system-of-systems approach?

The IDS will replace the Coast Guard's entire fleet of current deepwater surface and air assets. However, these assets are not being acquired on a one-for-one replacement basis. It is important to understand that the program takes a sophisticated, integrated approach to upgrade existing legacy assets while transitioning to newer more capable platforms. The acquisition strategy calls for the delivery of an entire system of interoperable platforms and supporting systems designed to meet performance-based requirements. The reason for focusing on a system-wide acquisition is to ensure compatibility and interoperability of deepwater assets, while providing high-levels of operational effectiveness and the most affordable solution for U.S. taxpayers.

DHS Strategic Goals

- From Sea-Air-Space 2005 presentation by RADM Patrick M. Stillman, Deepwater PEO
  – http://www.uscg.mil/deepwater/pdf/sea-air-space.pdf
- Network-centric SoS C4ISR enables and supports
  – Situation awareness
  – Seamless multi-service and civilian asset coordination
  – Prevention by deterrence and preemption
  – Engagement management

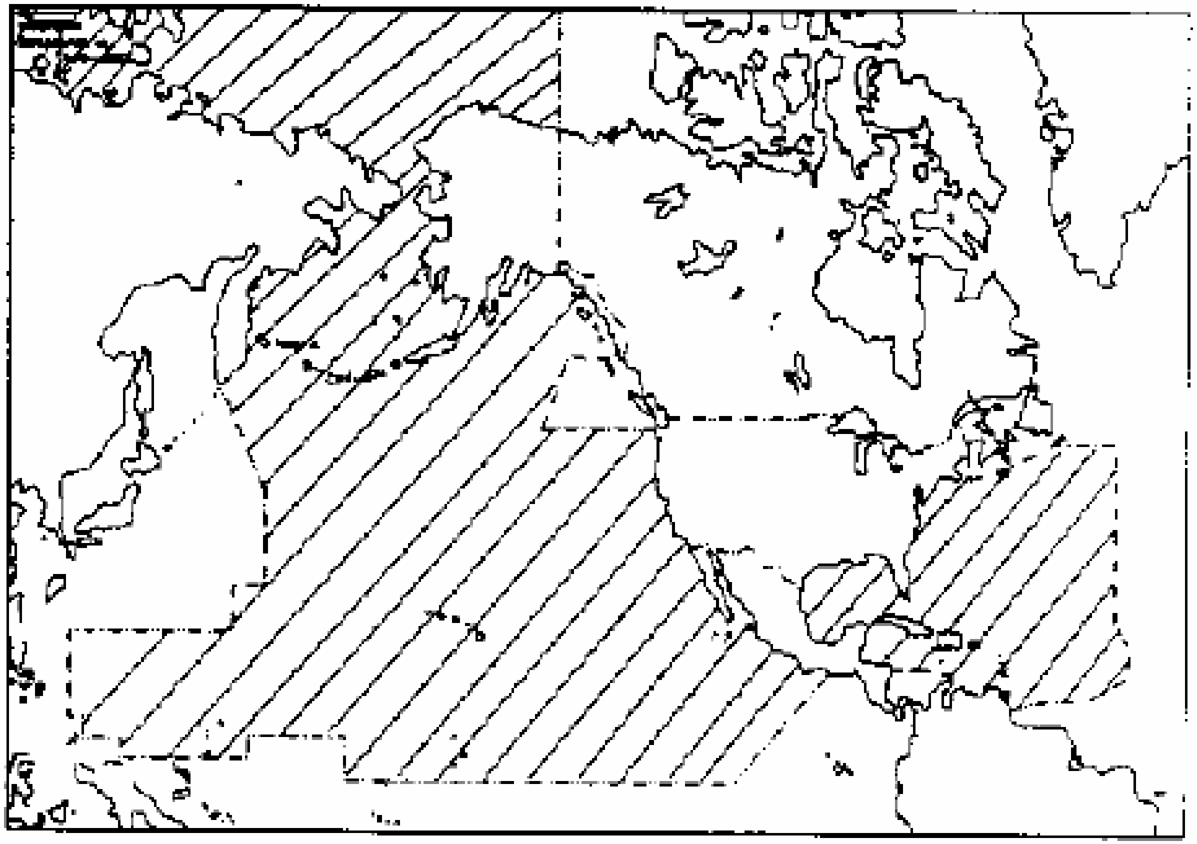
USCG Role

- USCG integrates homeland security (civilian assets) and homeland defense (DoD) assets and jurisdictions
  - USCG
  - Law enforcement, security and defense missions
  - Alien migratory interdiction missions
  - Support of military operations
- This defines a C4ISR capability requirement

Implicit C4ISR Requirements

- Sensors from disparate assets
  - Surface maritime radar
  - Surface-based aircraft radar and EO/IR
  - Land-based radar
  - Aircraft radar and EO/IR
  - Space-based sensors
- Sensor fusion issues include
  - Vast differences in data characteristics
  - Overlap and gaps in coverage

Deepwater Search-and-Rescue (SAR) Operational Area



Maritime SAR Region

Deepwater Assets

Deepwater C4ISR Vision

Spectrum of Fusion Requirements

- Cue assets
  - Cue EO/IR to radar targets for analysis
  - Interdiction of surface vessels “seen” by other platforms
- Handoff
  - Surface and air targets transition between target areas
  - Suveillance platform tracks to operational platforms
- Fusion
  - Situational awareness from overlapping sensor fields
  - Surface and air targets transitions
    - From coverage area to gaps
    - From gaps to other coverage areas

Sensor Fusion Challenges

- Situational awareness – multiple disparate sensors supporting central data bases
  - ForceNet data feeds to shore command centers
  - Radical differences in track characterizations
- Data formats
  - Coordinate systems – local non-inertial, ECIC, LLA…
  - Time offsets
  - Coordinate orientation calibration errors
  - Other biases

The Central Database

- A central database of object characteristics
  - Location of object
  - Characteristics of object
    - Type of object
    - Velocity, acceleration and turn rate
    - Track history
    - Proximity to other objects of local interest
    - Links to topical or awareness displays
- Available to multiple users

The Users

- Always online, all contacts
  - Civilian and law enforcement surveillance
  - Homeland defense situation awareness
- Operations users
  - USCG, USN, law enforcement, border patrol
  - Engagement support, theatre-wide and tactical
- Challenges
  - Selection of contacts relevant to each user
  - Appropriate depiction of contacts for user requirements

The DII COE Environment

- Defense Information Infrastructure
- Common Operating Environment
- A set of standards and support for a multi-service unified C4ISR capability
- Administered by the Defense Information Systems Agency (DISA)
- Starting point for research
  - http://www.dis.anl.gov/is/DIICOE.html

DII COE: A Framework for Interoperability

- Guidelines for
  - Software
    - Construction
    - Packaging
    - Behavior;
  - The operating environment;
  - The accompanying documentation
- Several compatible reference implementations of the operating environment for representative platforms, in the form of kernel and infrastructure support software

DII COE Support

- Guidelines for the reuse and sharing of software and data;
- A repository of shareable software and data;
- Tools and procedures for DII COE compliant mission applications
  - Submission and registration
  - Verification and certification

DII COE Segments

- Separately installable units of software and data
- The DII COE guidelines for software packaging prescribe
  - Kinds of segments;
  - The segment directory structure, which depends on the segment kind;
  - segment descriptor files, which convey the kind and other attributes of each segment, including information on the segment's dependence on other segments.

DII COE Reference Implementation

- Segment installer
  - A component of the COE kernel
  - Common across all DII COE platforms
  - Plug-and-play convenience
    - One element of the interoperability goal for mission application segments
    - The primary user of the information contained in segment descriptor files
  - Enforces segment dependencies.
- Accounts and profiles manager (APM)
  - Another component of the COE kernel
  - Administers certain operating system resources across a DII COE administrative domain on a local area network

Leading DII COE Implementations

- HARDPack
  - OEM is Lockheed Martin Federal Systems (LMFS), Owego, NY
  - Applications
    - USAF Airborne Warning and Control System (AWACS)
    - Regional / Sector Air Operations Center (R/SAOC) programs
- ORBexpress RT
  - OEM is Objective Interface Systems (OIS), Reston, VA
- The ACE ORB (TAO)
  - Open source product of Washington University, St. Louis, MO
  - Commercially supported by Object Computing, Inc., of St. Louis

All are CORBA Based SOAs

CORBA

- Common Object Request Broker Architecture
- Developed and administered by the Object Management Group (OMG)
- Platform-neutral architecture for combining heterogeneous software resources
- Basis for later Loosely Coupled Service Oriented Architectures (SOAs)

Topic 2: Estimation Theory

The Enabling Technology

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Estimation Theory Topics

- Defining the track data base
- Motion models and error propagation
- Data update and fusion
- Estimator families
- Square root filters – the SRIF
- Data fusion algorithmic architectures

The Track Data Base

- Which track data base
  - Platform sensors, platform track files
  - Tactical platform set, CIC tracks database
  - Theatre situation awareness, global data base
- Our interest today:
  - Global data base
  - Built from all available theatre sensors

The Theatre Track File Data Base

Theatre Track File

- The theatre track file
  - Is an object in a data base
  - Supports multiple users
  - Is maintained by algorithms based on estimation theory
- Track file users
  - Theatre wide situation awareness
  - Theatre engagement management
  - Tactical engagement management
  - Operations assets – situational alerting

Data Attributes of the Track

- Attributes include a superset of requirements of any one user
- Sometimes a few are missing
- Data attributes include
  - Track identifier
  - Contact type
    - Type (ship, aircraft, MIW, boat)
    - Specific ID (hull or tail number)
  - Position at epoch ("positime")
  - Motion model, velocity, and acceleration
  - Track quality and history
  - Other as users require

For Our Linear Motion Example

- The target position and motion state vector is

$$\underline{x} = \begin{bmatrix} \underline{p}(t) \\ \underline{v}(t) \\ \underline{a}(t) \end{bmatrix}$$

- A simple three degree of freedom motion model
- A six degree of freedom includes platform rotation – the three Euler angles or equivalent

Maintaining the Database

- Database attributes to be updated
  - Position
  - Motion model, velocity, acceleration
  - Epoch of latest update
  - Track quality and history
  - Type ID and other ancillary data
- Estimation theory supplies
  - Association of new data with proper track
  - Update of position and motion model
  - New estimates of track quality

Steps in Updating a Position Estimate

- Extract the initial data
  - Time of last update
  - Target position and motion model at last update
  - Track quality data after last update
- Extrapolate target data to time of new measurement
- Form new estimate of target data from extrapolated target data and new data

Basis of Motion Models

- Linear physics predicts position versus time
  - Position, velocity, and acceleration
  - Basic is the use of vector differential equations for equations of motion

$$\frac{d\underline{x}(t)}{dt} = \underline{f}(\underline{x}) + G \cdot \underline{w}$$

- Let's break this down so we can use it

The Motion Model

- The vector $\underline{x}(t)$
  - A structured collection of functions of time
  - Together they define the motion of the target
  - Elementary example – given position $\underline{p}$, velocity $\underline{vel}$, and acceleration $\underline{acc}$, we have

$$\underline{p}(t) = \underline{p}_0 + \underline{vel} \cdot t + \frac{1}{2} \cdot \underline{acc} \cdot t^2 + G \cdot \int_{t_0}^{t} \underline{w}(\tau) \cdot d\tau$$

- The vector $\underline{w}$ is the *process noise* with covariance $Q(t)$, uncorrelated in time

The Motion Model Matrices

- Define the matrices from the motion model

$$F(t) = \frac{\partial \underline{f}(\underline{x}(t))}{\partial \underline{x}(t)}$$

$$\frac{d\Phi(t)}{dt} = F(t), \Phi(t_0) = I \quad or \ \Phi(t) = \frac{\partial \underline{x}(t)}{\partial \underline{x}(t_0)}$$

$$\frac{dP(t)}{dt} = F(t) \cdot P(t) + P(t) \cdot F^T(t) + G \cdot Q(t) \cdot G^T$$

Summary of Motion Model Equations

Solving the Matrix Ricatti Equation

- The covariance extrapolation is

$$\frac{dP(t)}{dt} = F(t) \cdot P(t) + P(t) \cdot F^T(t) + G \cdot Q(t) \cdot G^T$$

- An approximation that omits terms quadratic in time is

$$\tilde{P}(t) = \Phi(t) \cdot P(t) \cdot \Phi^T(t)$$
$$+ G \cdot Q(t_0) \cdot G^T \cdot (t - t_0)$$

The Update and Fusion

- Vector $\underline{y}(\underline{x})$ of additional data with error covariance R
- Errors uncorrelated between updates
- Sensitivity matrix between state vectors H

$$H = \frac{\partial \underline{y}\left(\underline{x}\right)}{\partial \underline{x}}$$

- Result is minimum variance or MLE estimate based on old and new data

The Kalman Update

- The state vector update

$$K = P \cdot H^T \cdot R^{-1} = \tilde{P} \cdot H^T \cdot \left(H \cdot \tilde{P} \cdot H^T + R \right)^{-1}$$

$$\underline{\hat{x}} = \underline{\tilde{x}} + K \cdot \left(\underline{y} - \underline{h}\left(\underline{\tilde{x}} \right) \right)$$

- The covariance update

$$P = \left(I - K \cdot H \right) \cdot \tilde{P} = \left(I - K \cdot H \right) \cdot \tilde{P} \cdot \left(I - K \cdot H \right)^T$$

$$= \left(\tilde{P}^{-1} + H^T \cdot R^{-1} \cdot H \right)$$

Coordinate Systems

- Central database is usually kept in a global coordinate system, such as
  - ECIC (inertial), best for space objects
  - Latitude-longitude-altitude (LLA), best for surface objects, most aircraft
- Data from platforms may come in local coordinates
  - Local East-North-up (ENU) or North-East-down (NED)
  - Ship or platform coordinates
  - LLA may be used as a C2 convention

Estimation Challenges

- Association of updates with proper track files
- Local coordinates rotate with platform or Earth
- Offset or calibration errors introduce unobservable biases in the data
- One platform's new track object may be another platforms lost track object
- C2 data is often filtered, so data between updates is correlated
- Local track ID leaves global association of data to proper theatre-wide track ID to the data fusion methodology

Next: Our Toolbox

Track File Estimator Families

- Recursive (Kalman) filter family
  - Extended Kalman filter (EKF)
  - Recursive MLE (iterated EKF)
  - Square root filters – Potter, in “Astronautical Guidance,” R.H. Battin, Ed., McGraw-Hill (1964), currently available as ISBN 1-56-347342-9
  - Square root information filters – see “Factorizaton Methods for Discrete Sequential Estimation,” G.J. Bierman, Academic Press (1977) ISBN 0-12-097350-2 (also the best source for UDUT)
- Batch estimators
  - MLE, MAP
  - Least squares

Covariance Estimators

- The three Kalman updates given above
  - Based on linear error mapping equations
  - Most often used
- The particle filter
  - Monte Carlo methods
  - Most accurate but best application is study of simulations
- The unscented Kalman filter
  - Maps selected points that define the localization ellipsoid through nonlinearities
  - Practical accuracy approaches that of the particle filter
  - Practical for use in estimators

Summary of Motion Model Equations

Summary of Covariance Updates

- The Kalman updates
  – Best for simple trackers
  – Usable for most sensor fusion applications
- Square root filters
  – Similar in application to Kalman updates
  – Robust in practical tracking problems
- Particle and unscented Kalman filters
  – Best in highly nonlinear filtering applications
  – Should be considered for some sensor fusion applications

How Do We Meet the Challenges of Sensor Fusion?

- Filtered input data – use prewhitening input process
- Coordinate systems
  - Use ECIC or LLA for central data base
  - Translate in, translate out for DII COE
- For biases use consider states
  - Sometimes called the Carlson filter
  - Added states that model effects of unobservable motion model or update parameters such as biases

Prewhitening Correlated Measurements

- Consecutive correlated measurements $\underline{y}_{i\text{-}1}$ *and* $\underline{y}_{\mathrm{i}}$
- Make a new measurement $\underline{yu}_i$ by

$$\underline{yu}_i = \underline{y}_i - \rho \cdot \underline{y}_{i-1}$$

- Exact for Markov 1 processes
- New sequence of measurements will have minimal correlation

Batch Methods Can Prewhiten Intrinsically

What About Covariance Mapping in Sensor Fusion?

- First line of defense – Plan A
  - Use coordinate systems that map to LLA or ECIC without severe nonlinearities
  - Use Kalman covariance updates
- Plan B: Pose the fusion issue algebraically so that covariance errors are negligible as a practical matter
- Plan C: Use the unscented approach if necessary
- Plan D: Particle filtering approach

The Square Root Filters

- Algebraically identical to Kalman filters
- Numerically superior because
  - Covariance matrix is carried as a matrix with less dynamic range such as a Cholesky factor
  - Triangular matrices that are always positive semidefinite are used
- Other practical advantages for fusion
  - Ease of use with the algebraic problem statements in sensor fusion
  - Transparent use with disparate covariance matrices
- Types
  - Square root covariance filters such as the Potter
  - UDUT filter, a modified form of the square root covariance filter
  - Square root information filter, using the Normal Form for the Kalman update

Advantages and Disadvantages of Square Root Filters

- Square root covariance filters
  - Can be substituted for Kalman filters in the development process
  - But: Complex in formulation
- UDUT
  - Almost a drop-in for Kalman filters
  - Excellent numerical properties
  - But: Update data handled individually rather than as a vector
- SRIF
  - Excellent handling of poor observability and consider states
  - Closely related to normal form, MLE implementations
  - But: Substitution for Kalman filter requires some rewrite

Overview of the SRIF

- Square root information filter
- The [Fisher] information matrix is the inverse of the covariance matrix
- Based on posing extrapolation and update as normalized data equations (NDEs)of the form

$$A \cdot \Delta \underline{x} + \underline{b} = \underline{z}$$

- Covariance of both sides is normalized to *I*

Useful Properties of the NDE

- The NDE can be overdetermined
  - More rows than columns
  - New NDEs are constructed by augmenting smaller NDEs
- Annihilation of columns below the main diagonal preserves the NDE property
  - Left-multiply by a sequence of Householder transformations
  - Each operation preserves the NDE property
- Result is a square NDE and a decoupled trivial NDE
- The trivial NDE can be discarded
- The square NDE can be solved

Reduced NDE is the Enabling Technique for the SRIF

The Square Root Information Matrices

$$P^{-1} = R_x^T \cdot R_x$$

$$R^{-1} = R_v^T \cdot R_v$$

$$Q^{-1} = R_w^T \cdot R_w$$

Found by Cholesky Factorization or by SRIF

Use of an NDE for a Linear MLE

- Base overdetermined data equation

$$\underline{y} == \underline{h}\left(\underline{x}\right) + \underline{v}, \mathrm{Cov}\left\{\underline{v}\right\} = R = R_v^{-1} \cdot R_v^{-T}$$

- More rows than columns
- Converted to an NDE by left-multiplying by R_v

$$R_v \cdot \left(\underline{y} - \underline{h}\left(\underline{x}\right)\right) = \underline{z}_v = R_v \cdot \underline{v}_v$$

- Linearization provides a link to an estimate of $\underline{x}$

$$\underline{h}\left(\underline{x}\right) \approx \underline{h}\left(\underline{\hat{x}}\right) + H \cdot \left(\underline{x} - \underline{\hat{x}}\right)$$

- Solution is to solve for $\underline{x}$ as a recursion

Solving the MLE Equation

- Variable change

$$A \equiv R_v \cdot H$$

- Linearization of measurement equation

$$A \cdot \left(\hat{\underline{x}} - \underline{x} \right) + R_v \cdot \left(\underline{y} - \underline{h}\left(\hat{\underline{x}} \right) \right) = -\underline{z}_v$$

- Solution is the recursion

$$\hat{\underline{x}}_{i+1} = \hat{\underline{x}}_i + P \cdot A^T \cdot R_v \cdot \left(\underline{y} - \underline{h}\left(\hat{\underline{x}}_i \right) \right),$$

$$P^{-1} \equiv A^T \cdot A$$

A Simple Implementation

- Make the variable change

$$\underline{b}_i = R_v \cdot \left(\underline{y} - \underline{h}\left(\underline{\hat{x}} \right) \right)$$

- Now we have the simple overdetermined NDE

$$A \cdot \Delta \underline{x}_i + \underline{b}_i = \underline{z}$$

- We annihilate the columns of *A* below the main diagonal with a sequence of Householder transformations *T*

The Householder Transformation

- Basically a full rank matrix of the form
  $$H = I - 2 \cdot \underline{u} \cdot \underline{u}^T, \left| \underline{u} \right| = 1$$
- Properties as an operator on a vector
  - Characteristic values all +1 except a single -1
  - Characteristic vector $\underline{u}$ corresponds to the -1
  - As an operator, “reflects” vectors in the plane orthogonal to $\underline{u}$
  - Does not change the length of a vector
  - Each transform is symmetrical and orthogonal (idempotent)
- A matrix that is the product of Householder transformations
  - Orthogonal
  - In the abstract, represents a coordinate transformation

Annihilating a Column with a Householder Transformation

- Denote
  - The column of a matrix below the main diagonal as a vector $\underline{a}$
  - The element on the main diagonal as a_0
  - The length of $\underline{a}$ as a
- Construct a Householder transformation that results in a new column

Annihilating a Column with a Householder Transformation

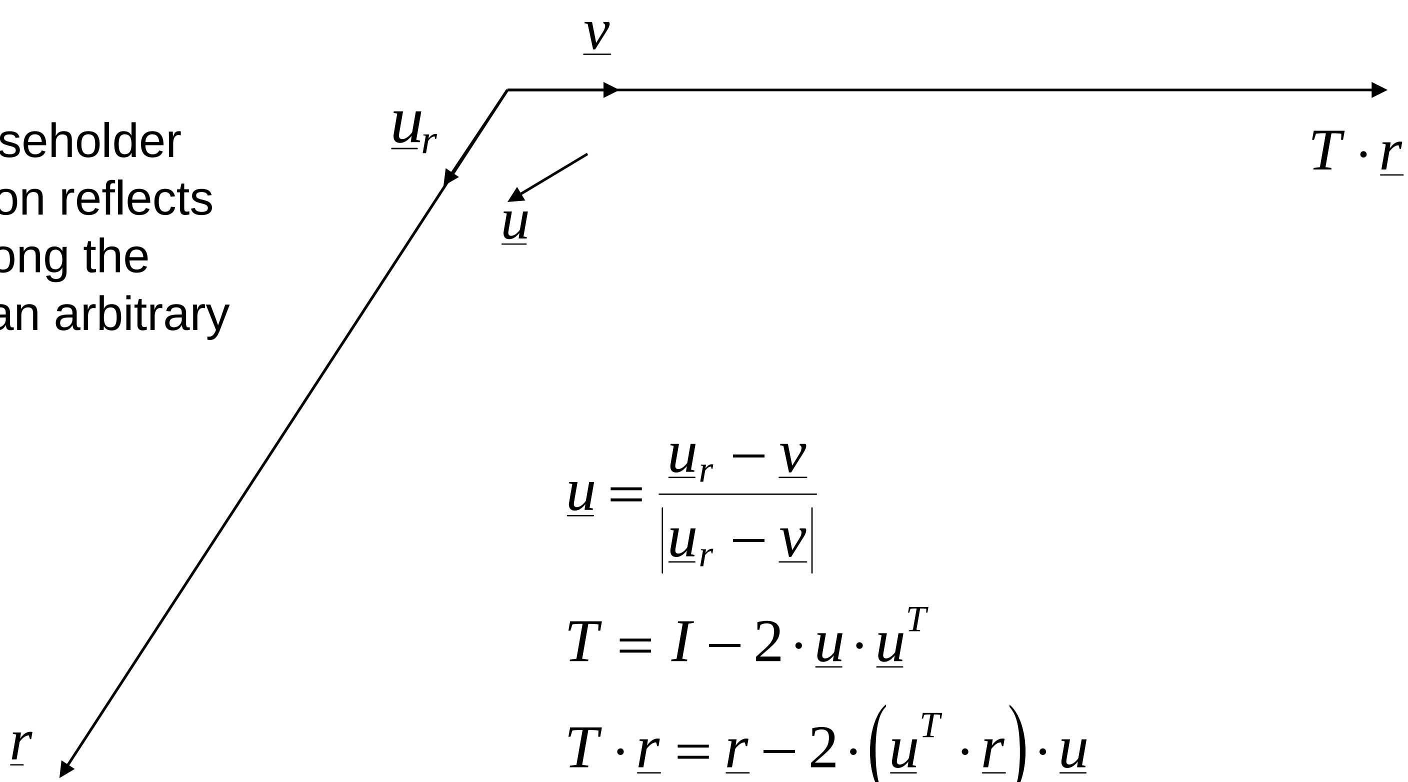
- The unit vector in the Householder transformation is

$$\underline{uu} = \underline{a} + \begin{bmatrix} \operatorname{sign}(a_0)\cdot\left|\underline{a}\right| \\ 0 \\ \vdots \end{bmatrix}, \left|\underline{uu}\right| = 2\cdot a\cdot\left(a + \left|a_0\right|\right)$$

- Sign is selected to give longest length of $\underline{uu}$

Use of Householder Transformation

Here, a Householder transformation reflects a vector r along the direction of an arbitrary vector v



$$\underline{u} = \frac{\underline{u}_r - \underline{v}}{\left|\underline{u}_r - \underline{v}\right|}$$

$$T = I - 2 \cdot \underline{u} \cdot \underline{u}^T$$

$$T \cdot \underline{r} = \underline{r} - 2 \cdot \left(\underline{u}^T \cdot \underline{r}\right) \cdot \underline{u}$$

Singularity?

- When a vector length *a* is zero
  - Means that a state is unobservable
  - True for consider states or belief model states
- What is done
  - Remove that state from the update
  - Preserve the numerical conditioning of the triangular matrix
- How it is done
  - Set the diagonal element to one
  - Set the rest of the row to zero

Result of Annihilation Operation

- The base operation

$$T \cdot A = \begin{bmatrix} R_x \\ 0 \end{bmatrix}$$

- The new NDE is

$$\begin{bmatrix} R_x \\ 0 \end{bmatrix} \cdot \Delta \underline{x}_i + \begin{bmatrix} \underline{e}_i \\ \underline{ze}_i \end{bmatrix} = \begin{bmatrix} \underline{z}_i{}' \\ \underline{ze}_i \end{bmatrix}$$

This is a Square Root Formulation

Canonic States

- With the square root formulation we are looking at a transformed state variable

$$\underline{zx} = R_x \cdot \underline{x}$$

- Properties of the new state vector $\underline{zx}$
  - States are uncorrelated
  - Variances are all unity

Numerical Properties are Ideal

SRIF Extrapolation

- State vector and covariance extrapolation are combined
- Actual state vector extrapolation may be by standard methods if only covariance extrapolation is needed
- Method
  - Construct a NDE from the error propagation equation
  - Operate on it with a sequence of Householder transformations

Posing the Process Noise as an NDE

- The process noise equation

$$\underline{w} = \underline{w}$$

- Covariance of both sides is Q
- Left multiplying by R_w normalizes the covariance of both sides to I
- The process noise NDE is

$$R_w \cdot \underline{w} = \underline{z}_w$$

Posing the Propagation of State Vector Errors as an NDE

- The linearized state vector propagation

$$\underline{\tilde{x}}(t) = \Phi \cdot \underline{\hat{x}}(t_0)$$

- The truth state vector propagation

$$\underline{x}(t) = \Phi \cdot \underline{x}(t_0) + G \cdot \underline{w}$$

- State vector error propagation

$$\left(\underline{x}(t) - \underline{\tilde{x}}(t)\right) = \Phi \cdot \left(\underline{x}(t_0) - \underline{\hat{x}}(t_0)\right) + G \cdot \underline{w}$$

SRIF Extrapolation NDE

- Normalizing and rearranging

$$-R_x\left(t_0\right)\cdot\Phi^{-1}\cdot G\cdot\underline{w}+R_x\left(t_0\right)\cdot\Phi^{-1}\cdot\Delta\underline{\tilde{x}}=\underline{\tilde{z}}_x$$

$$\underline{\tilde{z}}_x=R_x\left(t_0\right)\cdot\left(\underline{x}\left(t_0\right)-\underline{\hat{x}}\left(t_0\right)\right),\Delta\underline{\tilde{x}}=\left(\underline{x}\left(t\right)-\underline{\tilde{x}}\left(t\right)\right)$$

- Augmenting these NDEs gives us

$$\begin{bmatrix} R_w & 0 \\ -R_x\left(t_0\right)\cdot\Phi^{-1}\cdot G & R_x\left(t_0\right)\cdot\Phi^{-1}\end{bmatrix}\cdot\begin{bmatrix}\underline{w}\\ \Delta\underline{\tilde{x}}\end{bmatrix}=\begin{bmatrix}\underline{z}_w\\ \underline{\tilde{z}}_x\end{bmatrix}$$

The Triangularized NDE

- The base operation

$$\tilde{T} \cdot \begin{bmatrix} R_w & 0 \\ -R_x\left(t_0\right)\cdot\Phi^{-1}\cdot G & R_x\left(t_0\right)\cdot\Phi^{-1} \end{bmatrix} = \begin{bmatrix} \tilde{R}_x \\ 0 \end{bmatrix}$$

- The result

$$\begin{bmatrix} \tilde{R}_w & \tilde{R}_{wx} \\ 0 & \tilde{R}_x \end{bmatrix} \cdot \begin{bmatrix} \underline{w} \\ \Delta\underline{\tilde{x}} \end{bmatrix} = \begin{bmatrix} \underline{zw} \\ \underline{zx} \end{bmatrix}$$

- We extrapolate both the data and covariance

Submatrices of the Annihilation Transformation

$$\tilde{T} = \begin{bmatrix} \tilde{R}_w \cdot R_w^{-1} + \tilde{R}_{wx} \cdot G & \tilde{R}_{wx} \cdot \Phi \cdot R_x^{-1}\left(t_0\right) \\ \tilde{R}_x \cdot G \cdot R_w^{-1} & \tilde{R}_x \cdot \Phi \cdot R_x^{-1}\left(t_0\right) \end{bmatrix}$$

$$\begin{aligned} \tilde{T}_{11} \cdot \tilde{T}_{11}^T &= I - R_w^{-T} \cdot G^T \cdot \tilde{R}_x^T \cdot \tilde{R}_x \cdot G \cdot R_w^{-1} \\ &= \left[I + R_w^{-T} \cdot G^T \cdot \Phi^{-T} \cdot R^T\left(t_o\right) \cdot R\left(t_o\right) \cdot \Phi^{-1} \cdot G \cdot R_w^{-1} \right] \end{aligned}$$

$$\tilde{T}_{12} \cdot \tilde{T}_{12}^T = I - R_x^{-T}\left(t_0\right) \cdot \Phi^T \cdot \tilde{R}_x^T \cdot \tilde{R}_x \cdot \Phi \cdot R_x^{-1}\left(t_0\right)$$

The SRIF Data Update

- Very simple
  - Combine extrapolated states and covariances with new data
  - Pose as data equation and triangularize as with Householder transformations
- The data equations as NDEs are

$$R_x \cdot \Delta \underline{x} = \underline{zx}$$

$$R_v \cdot \left(\underline{y} - \underline{h}\left(\underline{x} \right) \right) = \underline{zv}$$

Augmented Update NDE

- Make the variable change

$$\underline{b} = R_v \cdot \left(\underline{y} - \underline{h}\left(\hat{\underline{x}}\right)\right)$$

- With the simple overdetermined NDE

$$A \cdot \Delta\underline{x} + \underline{b} = \underline{z}$$

- We augment the NDEs to form

$$\begin{bmatrix} \tilde{R}_x \\ A \end{bmatrix} \cdot \Delta\underline{x} + \begin{bmatrix} \underline{0} \\ \underline{b} \end{bmatrix} = \begin{bmatrix} \underline{zv} \\ \underline{z} \end{bmatrix}$$

Implementing the SRIF Update

- Triangularize the augmented data equation
- The base equations are

$$T \cdot \begin{bmatrix} \tilde{R}_x \\ A \end{bmatrix} = \begin{bmatrix} R_x \\ 0 \end{bmatrix}, \; T \cdot \begin{bmatrix} \underline{0} \\ \underline{b} \end{bmatrix} = \begin{bmatrix} \underline{e} \\ \underline{zu} \end{bmatrix}$$

- The solution is

$$\begin{bmatrix} \tilde{R} \\ 0 \end{bmatrix} \cdot \Delta \underline{x} + \begin{bmatrix} \underline{e} \\ \underline{zu} \end{bmatrix} = \begin{bmatrix} \underline{zx} \\ \underline{zu} \end{bmatrix}$$

Submatrices of the Annihilation Transformation

$$T = \begin{bmatrix} R_x^{-T} \cdot \tilde{R}^T & R_x^{-T} \cdot A^T \\ T_{21} & T_{22} \end{bmatrix}$$

$$T_{21} \cdot T_{21}^T = I - \tilde{R}_X \cdot R_x^{-1} \cdot R_x^{-T} \cdot \tilde{R}_x^{-T}$$

$$\begin{aligned} T_{22} \cdot T_{22}^T &= I - A \cdot R_x^{-1} \cdot R_x^{-T} \cdot A^T \\ &= \left[I + A \cdot \tilde{R}_x^{-1} \cdot \tilde{R}_x^{-T} \cdot A^T \right] \end{aligned}$$

Summary

- The EKF methods
  - Traditional EKF, with the Joseph stabilized form
  - SRIF, UDUT, Potter are the big hammers
- Batch methods
  - MLE/MAP
  - Least squares
- Unscented or particle filter covariances
  - May be needed when strong nonlinearities are unavoidable
  - May be used for measurement or extrapolated state covariances

Capabilities and Remarks

	EKF	SRIF	UDUT
Covariance Stability	Good with Josephson stabilized form	Excellent	Excellent
State extrapolation	Provide externally	Provide externally	Provide externally
Iterated/MLE	Recompute H, P, K for each iteration	Recompute H for each iteration	One measurement at a time
Particle/Unscented	Use for R, extrapolated P	Use for R, replace SRIF’s extrapolation of P	Use for R, replace extrapolation of P
Belief/Consider States	Transparent, zero rows of K matrix	Transparent	Transparent, zero rows of K vectors

Batch vs. Recursive

	Recursive	MLE/MAP	Least Squares
Process noise	YES	NO	NO
Cramer-Rao bound	NO	Asymptotic	Near
Biased estimates	YES	Asymptotically unbiased	NO
Covariance estimate	Typically 5X	YES	NO

Typical Track Maintenance Architecture

- Base trackers are two-state
  - Range and range rate, etc.
  - Very simple EKFs
  - Function is to support association
- Estimation trackers are 3-DOF or 6-DOF
  - EKF or square root filter
  - Initialized from data history and EKF or least squares
  - Function is to support main tracker requriements
- Highest performance estimators are batch
  - MLE/MAP
  - Initialized by EKF, least squares, or binary data splitting

Binary Data Splitting

- Used to initialize MLEs when a large amount of data is present
- Deals with numerical issues of nonlinear recursion for this problem
- Technique begins with first, last, and center data point in data history, implement MLE
- Process proceeds
  - Add data points midway between existing points and update MLE
  - Continue until the entire measurement history is included

Topic 3: Data Fusion from Multiple Sensors

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Topics

- Problem definition
  - Multiple sensors
  - Multiple origins
- Challenges
  - Association of data to fused tracks
  - Dropouts in space and time
  - Gridlocking and biases
  - Correlated data from sequential tracker outputs
- Solutions

Problem Definition

- The good
  - Merged data provides higher accuracy and greater reliability
  - Fused data is available to multiple users
- The bad
  - Track data must be
- The ugly
  - Biases, dropouts, ambiguities can cause corruption of the database
  - Best recovery may be rebuilding

Merging Data from Multiple Platforms

- Advantages
  - Triangulation provides better accuracy
  - Overlapping sensor fields reduce dropouts
- Disadvantages
  - Ambiguities act like false returns
  - Can generate false tracks
  - Association of data to the proper track file requires is linked to the gridlocking problem and bias estimation

Basic Issue

- Contributions to database
  - Usually from trackers on platforms
  - Data is kept to a minimum to reduce bandwidth usage
  - Can consist of filtered or smoothed data

- Problems
  - Covariance data and epoch is summarized or absent
  - State estimates from filters have correlated errors

C2 Data Requirements

- Data should include
  - States, epoch, last raw measurement, and covariances
  - Platform position and coordinate system data
- Data fusion platform often must solve problem with insufficient information
  - Covariance estimated from first principles
  - Additional state augmentation to accommodate missing information and correlated inputs

The Localization Ellipse

- A measure of how accurate the state estimates are
- Based on the vector Gaussian PDF

$$p\left(\underline{x}\right)=\frac{1}{\left|P\right|^{1/2}\cdot\left(2\pi\right)^{N/2}}\cdot\exp\left(-\frac{1}{2}\cdot\Delta\underline{x}^{T}\cdot P^{-1}\cdot\Delta\underline{x}\right)$$

- Equation defines a “one-sigma” ellipsoid

$$\Delta\underline{x}^{T}\cdot P^{-1}\cdot\Delta\underline{x}=1$$

In Sensor Fusion

- The covariance matrix for fused data from source 1 and source 2 can be posed as

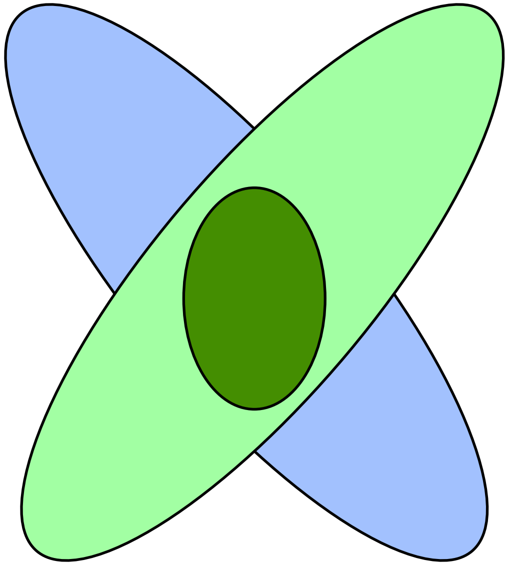
$$P^{-1} = P_1^{-1} + P_2^{-1}$$

- The localization ellipsoid for the fused data is

$$\Delta\underline{x}^T \cdot P^{-1} \cdot \Delta\underline{x} = \Delta\underline{x}^T \cdot \left(P_1^{-1} + P_2^{-1}\right) \cdot \Delta\underline{x} = 1$$

- This shows that the localization ellipsoid for fused data is wholly contained within the localization ellipsoid of each input data set

Two Radars, One Target



- Radar 1 (Blue)

$$p_1(\underline{x}) = \frac{1}{(2\pi)^{\frac{N}{2}} \cdot |P_1|^{\frac{1}{2}}} \cdot \exp\left(-\frac{1}{2}(\underline{x} - \underline{x}_1)^T \cdot P_1^{-1} \cdot (\underline{x} - \underline{x}_1) \right)$$

- Radar 2 (Lime Green)

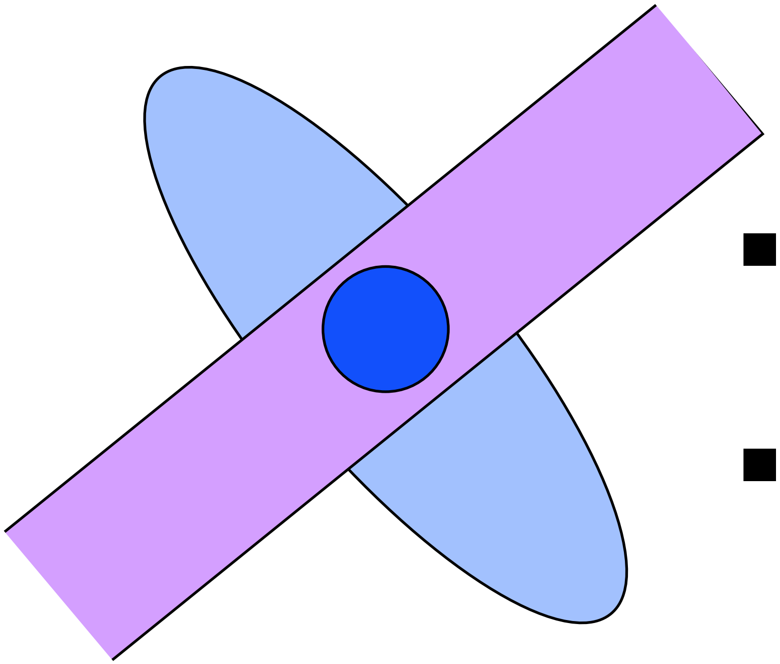
$$p_2(\underline{x}) = \frac{1}{(2\pi)^{\frac{N}{2}} \cdot |P_2|^{\frac{1}{2}}} \cdot \exp\left(-\frac{1}{2}(\underline{x} - \underline{x}_2)^T \cdot P_2^{-1} \cdot (\underline{x} - \underline{x}_2) \right)$$

- Combined (Green)

$$P_3^{-1} = P_1^{-1} + P_2^{-1}$$

$$\underline{x}_3 = P_3 \cdot \left(P_1^{-1} \cdot \underline{x}_1 + P_2^{-1} \cdot \underline{x}_2 \right)$$

Radar with EO/IR or Passive Sonar on Separate Platform



- Radar 1 (Light Blue)
- EO/IR or Passive Sonar (Violet)
- Combined (Dark Blue)

A Classical MLE Example

- Simple one-dimensional, constant velocity

$$s(t) = s_0 + sv \cdot (t - t_0)$$

- A set of *M* position measurements

$$y_i = s(t_i) + n_i, \mathrm{var}(n_i) = \sigma_n^2$$

- Vector notation

$$\underline{y} = H \cdot \underline{x} + \underline{v}, \mathrm{cov}(\underline{v}) = R = \sigma_n^2 \cdot I$$

Developing the MLE

- The measurement sensitivity matrix H and state vector $\underline{x}$

$$H = \begin{bmatrix} 1 & t_1 - t_0 \\ 1 & t_2 - t_0 \\ \vdots & \vdots \\ 1 & t_M - t_0 \end{bmatrix}, \ \underline{x} = \begin{bmatrix} s_0 \\ sv \end{bmatrix}, \ t_0 = \text{measurement epoch}$$

- The likelihood function

$$p\left(\underline{y} \middle| \underline{x}\right) = \frac{1}{\sigma_n^M \cdot (2\pi)^{M/2}} \cdot \exp\left(-\frac{1}{2} \cdot \frac{\left(\underline{y} - H \cdot \underline{x}\right)^T \cdot \left(\underline{y} - H \cdot \underline{x}\right)}{\sigma_n^2} \right)$$

Solving the MLE

- The log likelihood function

$$l\left(\underline{x}\right)=-\frac{M}{2}\cdot\ln\left(\sigma_n^2\right)-\frac{M}{2}\cdot\ln\left(2\pi\right)-\frac{1}{2\sigma_n^2}\cdot\left(\underline{y}-H\cdot\underline{x}\right)^T\cdot\left(\underline{y}-H\cdot\underline{x}\right)$$

- The likelihood equation

$$\frac{\partial l\left(\underline{x}\right)}{\partial\underline{x}}=\frac{1}{\sigma_n^2}\cdot H^T\cdot\left(\underline{y}-H\cdot\underline{\hat{x}}\right)=\underline{0}$$

- The estimator and its error covariance

$$\underline{\hat{x}}=P\cdot H^T\cdot R^{-1}\cdot\underline{y}=\left(H^T\cdot H\right)^{-1}\cdot H^t\cdot\underline{y}$$

$$P=\left(H^T\cdot H\right)^{-1}\cdot\sigma_n^2$$

Simplified Notation

- Notation

$$\left\langle t \right\rangle \equiv \frac{1}{M} \cdot \sum_{i=1}^{M} t_i , \left\langle t^2 \right\rangle \equiv \frac{1}{M} \cdot \sum_{i=1}^{M} t_i^2$$

$$\left\langle y \right\rangle \equiv \frac{1}{M} \cdot \sum_{i=1}^{M} y_i , \left\langle y \cdot t \right\rangle = \frac{1}{M} \cdot \sum_{i=1}^{M} y_i \cdot t_i ,$$

- Simplified forms

$$\sigma_t^2 = \left\langle t^2 \right\rangle - \left\langle t \right\rangle^2 , \; dt = \left\langle t_0 - t \right\rangle = t_0 - \left\langle t \right\rangle$$

$$\left\langle \left(t - t_0 \right)^2 \right\rangle = \left\langle t^2 \right\rangle - 2 \cdot \left\langle t \right\rangle \cdot t_0 + t_0^2 = \sigma_t^2 + dt^2$$

The Matrices

- The simplified notation

$$\frac{1}{\sigma_n^2} \cdot H^T \cdot H = P^{-1} = \frac{M}{\sigma_n^2} \cdot \begin{bmatrix} 1 & -dt \\ -dt & \sigma_t^2 + dt^2 \end{bmatrix}$$

$$H^T \cdot \underline{y} = M \cdot \begin{bmatrix} \langle y \rangle \\ \langle y \cdot t \rangle - \langle y \rangle \cdot t_0 \end{bmatrix}$$

- Sets the stage for the solution

Looking at the Solution

$$\underline{\hat{x}} = \begin{bmatrix} s_0 \\ sv \end{bmatrix} = \begin{bmatrix} \langle y \rangle + sv \cdot \left(t_0 - \langle t \rangle \right) \\ \dfrac{\langle y \cdot t \rangle - \langle y \rangle \cdot \langle t \rangle}{\sigma_t^2} \end{bmatrix}$$

$$P = \frac{\sigma_n^2}{M \cdot \sigma_t^2} \cdot \begin{bmatrix} \sigma_t^2 + dt^2 & dt \\ dt & 1 \end{bmatrix}$$

Square Root Matrices

$$P^{-1} = R_x^T \cdot R_x$$

$$R_x = \sqrt{\frac{M}{\sigma_n^2}} \cdot \begin{bmatrix} 1 & -dt \\ 0 & \sqrt{\sigma_t^2} \end{bmatrix}$$

$$P = R_x^{-1} \cdot R_x^{-T}$$

$$R_x^{-1} = \sqrt{\frac{\sigma_n^2}{M \cdot \sigma_t^2}} \cdot \begin{bmatrix} \sqrt{\sigma_t^2} & dt \\ 0 & 1 \end{bmatrix}$$

Key Interpretations

- Position estimate is linear combination of the mean of the measurements and the estimated velocity
- Neither the velocity estimate nor its variance is a function of the epoch
- The variance of the position estimate is quadratic in the deviation of the epoch from the center of the estimation interval <t>
- More to follow

Special Case: Uniform Time Intervals

$$t_i = i \cdot \Delta T$$

$$\langle t \rangle = \frac{M+1}{2} \cdot \Delta T, \langle t^2 \rangle = \frac{(M+1)\cdot(2M+1)}{6} \cdot (\Delta T)^2$$

$$\sigma_t^2 = \langle t^2 \rangle - \langle t \rangle^2 = \frac{1}{12} \cdot (M^2 - 1) \cdot (\Delta T)^2$$

$$P = \frac{\sigma_n^2}{M} \cdot \begin{bmatrix} 1 + \frac{dt^2}{\sigma_t^2} & \frac{dt}{\sigma_t^2} \\ \frac{dt}{\sigma_t^2} & \frac{1}{\sigma_t^2} \end{bmatrix}$$

Covariance Matrix

$$P = \frac{\sigma_n^2}{M} \cdot \begin{bmatrix} 1 + \frac{12 \cdot \left(t_0 - \langle t \rangle\right)^2}{\left(M^2 - 1\right) \cdot \left(\Delta T\right)^2} & \frac{12 \cdot \left(t_0 - \langle t \rangle\right)}{\left(M^2 - 1\right) \cdot \left(\Delta T\right)^2} \\ \frac{12 \cdot \left(t_0 - \langle t \rangle\right)}{\left(M^2 - 1\right) \cdot \left(\Delta T\right)^2} & \frac{12}{\left(M^2 - 1\right) \cdot \left(\Delta T\right)^2} \end{bmatrix}$$

Estimation at End of Interval

$$t_0 - \langle t \rangle = \frac{M-1}{2} \cdot \Delta T$$

$$P = \frac{2 \cdot \sigma_n^2}{M \cdot (M-1)} \cdot \begin{bmatrix} 2 \cdot M + 1 & \dfrac{6}{\Delta T} \\ \dfrac{6}{\Delta T} & \dfrac{6}{(M+1) \cdot (\Delta T)^2} \end{bmatrix}$$

Cholesky Factor of Covariance Matrix

$$R_x^{-1} = \frac{\sigma_n}{\sqrt{M}} \cdot \begin{bmatrix} 1 & \frac{t_0 - \langle t \rangle}{\Delta T} \cdot \sqrt{\frac{12}{M^2 - 1}} \\ 0 & \sqrt{\frac{12}{\left(M^2 - 1\right) \cdot \left(\Delta T\right)^2}} \end{bmatrix}, \; P = R_x^{-1} \cdot R_x^{-T}$$

Square Root Information Matrix

$$R_x = \frac{\sqrt{M}}{\sigma_n} \begin{bmatrix} 1 & -\left(t_0 - \langle t \rangle\right) \\ 0 & \sqrt{\dfrac{\left(M^2 - 1\right) \cdot \left(\Delta T\right)^2}{12}} \end{bmatrix}, \quad P^{-1} = R_x^T \cdot R_x$$

Area of Localization Ellipse

- Area of localization ellipse is

$$A_e = \pi \cdot a \cdot b = \pi \cdot \sqrt{\lambda_1 \cdot \lambda_2} = \pi \cdot |P|^{\frac{1}{2}}$$

$$= \frac{\pi \cdot \sigma_n^2}{M \cdot \sigma_t} = \frac{\pi \cdot \sigma_n^2}{M \cdot \Delta T} \cdot \sqrt{\frac{12}{M^2 - 1}}$$

- Varies inversely as M^2, or integration time and number of measurements

Information Not a Function of Epoch t_0

A Common Practical Problem

- Condition number of covariance matrix
  - Proportional to square of integration time
  - Easily seen in our example for estimation at any epoch
- Process noise limits covariance condition number
  - Problem is bound for this type of system model
  - Commonly used to bound problem in zero process noise system models
  - Process noise bounds convergence of estimate and thus accuracy and performance of tracks – and merged data

Our Example

- Covariance matrix condition number
- Condition number is asymptotically
  - Estimation of position at center of interval

$$\frac{\lambda_2}{\lambda_1} \sim \frac{1}{12} \cdot M^2$$

  - Estimation at time of last measurement

$$\frac{\lambda_2}{\lambda_1} \sim \frac{4}{3} \cdot M^2$$

Covariance Collapse

- A tracker problem caused by high accuracy of some states relative to that of others
- Covariance condition number is an indicator
- Typified by accuracy of velocity versus position estimates for non-accelerating target
  - Position variance approaches a limit
  - Velocity variance decreases by $1/T^2$
  - Covariance matrix condition number increases as T^2
- Can also be a problem with multiple platform fused tracks

What Estimator to Use?

Estimator	Advantages	Disadvantages
EKF	Simple to apply	Others perform better
SRIF	• Full transparency in operation • Shares Householder annihilation with MLE/MAP • Can be implemented with zero process noise	Initial startup is more complex
UDUT	• Drop-in replacement for EKF	• Updates are one scalar measurement at a time • Efficiency issues with association checks in dense environment
Batch MLE/MAP	• Best accuracy – asymptotically achieves the Cramer-Rao Bound, asymptotically unbiased • Can be used to initialize EKF/SRIF/UDUT	Requires finesse to start up with large bodies of data
Batch least squares	• Simple to apply and use • Good statistical efficiency • Simpler than MLE/MAP • Can be used to initialize EKF/SRIF/UDUT or MLE/MAP	No implicit estimation of covariance

Use of Estimators

- Surface vessels, other slow movers
  - Simple four-state
  - EKF is sufficient
- Submarines, orbital, and other zero process noise contacts
  - Transitions – EKF to SRIF to MLE/MAP
  - Re-initialize with batch least squares
- High performance contacts – as needed

Bring Them All to the Table

Advanced Techniques

- Concatenation of state vectors of multiple tracks (Bar-Shalom)
  - Account for probability of misassociation in correlation between tracks
  - Account for same data updating multiple tracks as correlation between tracks
- Track-before-detect
  - Build log likelihood of a contact versus position
  - Allow probability of misassociation to build
  - An alternative interpretation of MHT

Meeting Needs of Disparate Users with the Track Database

- The central track files are the data base
- Attributes of each track include
  - States, covariances, epoch of last update
  - Other quality of track parameters
  - Type and ID information
  - Status such as C2 connectivity, sensor suite…
- User outputs are reports from the data base

Topic 4: C4ISR Architectural Issues

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Topics

- The association problem
- Dropouts
- Architecture trade space
  - Number of hosts – distributed vs. centralized
  - Raw data versus filtered data
  - RF Bandwidth usage

The Association Problem

- The greatest challenges
  - Biases and gridlocking
  - Ambiguities
  - Computation loading proportional to number of inputs times number of tracks
- One approach
  - Batch inputs and extrapolate to single epoch
  - Batch updates and threshold on Bhatacharya distance
- Use tracker techniques as required
  - MHT to track ambiguities without reporting them
  - IMM to allow for different possibilities of target type and state

Coverage Dropouts

- Platforms will transition between coverage areas over the theatre
- Situation awareness QOS improved by properly associating tracks
- One approach
  - Use ID data in tracks
  - Apply dead reckoning to supply candidates of old tracks for new contacts
  - Use sequential likelihood threshold of ID characteristics to associate new tracts with old tracks

An Approach to the Dropout Problem

- Use dead reckoning candidates as track data base attributes for information only
- Hard associate only on
  - Satisfaction of track association criteria such as Bhatacharya distance

$$d_{bhat}^2 = \left(\Delta \underline{x}\right)^T \cdot \left(P_1^{-1} + P_2^{-1}\right) \cdot \left(\Delta \underline{x}\right)$$

  - Matchup of specific platform identification

Architecture Trade Space: Number of Hosts

- Basic architecture decision: Centralized vs. distributed fusion databases
- Trades and impacts
  - Latency to users
  - RF bandwidth usage
  - Robustness of network
- Reliability
  - Single point failure mode vs. soft casualty modes
  - “The man with one watch knows what time it is – the man with two watches is never sure.”

Latency to Users

- Centralized databases
  - Some users will be several links away from the host
  - High priorities in C2 bandwidth allocation can increase latencies
- Distributed databases
  - Users can select closest host
  - Tactical links have fewer priority gridlocks than strategic links

Bandwidth Usage

- Centralized databases
  - All the users will be sent data from the central database
  - All the sensors will send data to the central database
- Distributed databases
  - Each database gets only the data it needs for its users
  - Each database sends data to local users

Robustness of Network

- Centralized databases
  - Single-point failure in C2 or host requires backup
- Distributed databases
  - Loss of a node or C2 link has only local impact
  - “The man with one watch knows what time it is – the man with two watches is never sure.”

Common Sense Solution

- Use both
  - Centralized databases for strategic data users
  - Theatre databases for tactical users
  - Equip every CIC for sensor fusion
- Minimization of bandwidth usage
  - Theatre databases support strategic databases
  - Backup is data directly from platforms
  - Bandwidth-adaptive data format
    - Full track file data with last measurement when C2 bandwidth allows
    - Only data, coordinates, and epoch when necessary to get through
    - Variable formats supported by DII/COE/CORBA/SOAs

Recommended Reference

- Handbook of Multisensor Data Fusion, edited by David L Hall and James Llinas
- Chapters include
  - Multisensor Data Fusion – David Hall and James Llinas
  - Algorithmics of Data Association in Multiple-Target Tracking – Jeffrey K Uhlmann (also a more advanced chapter)
  - Data Registration – Richard R Brooks and Lynne Grewe
  - Target Tracking in Sonar, Radar, and EO/IR – T. Kirubarajan and Yaakov Bar-Shalom
  - Data Fusion in Nonlinear Systems – Simon Julier and Jeffrey K Uhlmann
  - Requirements Derivation for Data Fusion Systems – Ed Waltz and David L Hall
  - Dirty Secrets in Multisensor Data Fusion – David L Hall and Alan N Steinberg
  - A Survey of Multisensor Data Fusion Systems – Mary L Nichols

Useful Books

- "Handbook of Multisensor Data Fusion" by David L. Hall and James Llinas, CRC (2001) ISBN 0-8493-2397-7 (see previous slide)
- "Design and Analysis of Modern Tracking Systems" by Sam Blackman and Bob Popoli, Artech (1999) ISBN 1-5805-3006-0
- "Multiple Target Tracking in a Dense Environment," S. Blackman, Artech (1988) ISBN 0-8900-6179-3

Useful Books

- "Applied Optimal Estimation," A. Gelb, Ed. MIT Press, (1974) ISBN 0-2625-7048-3
- "Factorization Methods in Discrete Sequential Estimation," G. Bierman, Academic Press (1977) ISBN 0-1209-7350-2
- "Spectral Analysis and Time Series," M. B. Priestly, Academic Press (1989) ISBN 0-1256-4921-5

The End